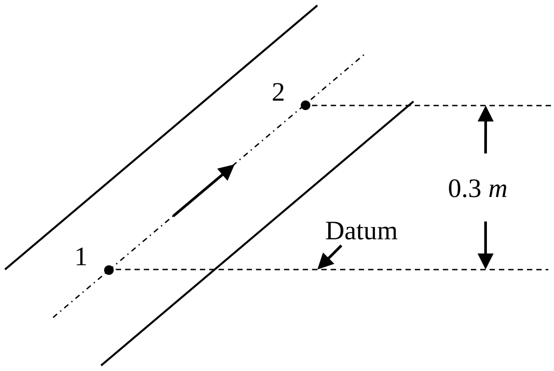
ENERGY EQUATION (APPLICATIONS)

Example. Water flows through a pipeline of constant diameter that is inclined upwards. On the centreline of the pipe, point 1 is 0.3 *m* vertically below point 2. The pressure at point 1 is 9300 $N\ m^{-2}$. What is the pressure at point 2 if there is no loss of energy?

Solution.



Let us assume the datum to pass through the lower point 1. Hence, datum head at point 1, $Z_1 = 0$. As point 2 is located above point 1 at a vertical height of 0.3 m, we have datum head at point 2, $Z_2 = 0.3\ m$. Let the mean velocity of flow at points 1 and 2 be V_1 and V_2 respectively. As the pipeline is of constant diameter, the mean velocity of flow at each point on the centreline of the pipe must be the same. That is, mean velocity of flow at point 1, V_1 = mean velocity of flow at point 2, V_2. Let the pressure intensity at point 1 and 2 be p_1 and p_2 respectively.

Applying Bernoulli's equation between points 1 and 2, we have,

$$Z_1 + \frac{p_1}{\gamma_w} + \frac{V_1^2}{2g} = Z_2 + \frac{p_2}{\gamma_w} + \frac{V_2^2}{2g}$$

As $V_1 = V_2$, we have, $\frac{V_1^2}{2g} = \frac{V_2^2}{2g}$. Therefore, the velocity heads get cancelled and the Bernoulli's equation written above becomes

$$Z_1 + \frac{p_1}{\gamma_w} = Z_2 + \frac{p_2}{\gamma_w}$$

$$\Rightarrow 0 + \frac{9300}{9810} = 0.3 + \frac{p_2}{9810}$$

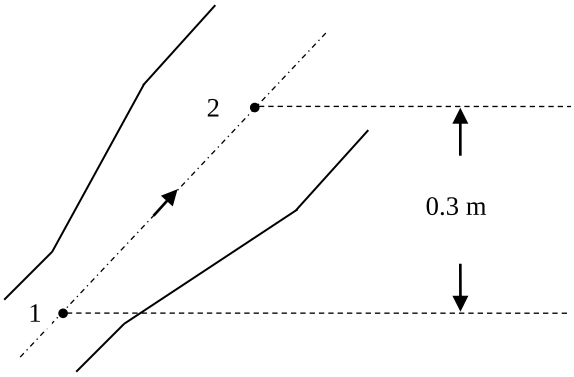
$$\Rightarrow 0.948 = 0.3 + \frac{p_2}{9810}$$

$$\Rightarrow \frac{p_2}{9810} = 0.948 - 0.3 = 0.648$$

$$\Rightarrow p_2 = 0.648 \text{ x } 9810 = 6357 \; N\, m^{-2}$$

Example. Water flows through an expanding pipeline that is inclined upwards. On the centreline of the pipe, point 1 is 0.3 *m* below point 2. The velocities are $V_1 = 3.1\ m\ s^{-1}$ and $V_2 = 1.7\ m\ s^{-1}$. The pressure at point 1 is $9.3 \text{ x } 10^3\ N\ m^{-2}$. What is the pressure at point 2 if there is no loss of energy?

Solution.



Assuming the datum to pass through the lower point 1, we have, datum head at point 1, $Z_1 = 0$ and datum head at point 2, $Z_2 = 0.3$ m.

Applying Bernoulli's equation between points 1 and 2, we have,

$$Z_1 + \frac{p_1}{\gamma_w} + \frac{V_1^2}{2g} = Z_2 + \frac{p_2}{\gamma_w} + \frac{V_2^2}{2g}$$

$$\Rightarrow 0 + \frac{9.3 \text{ x} 10^3 \text{ N m}^{-2}}{9810 \text{ N m}^{-3}} + \frac{(3.1 \text{ m s}^{-1})^2}{2 \text{ x} 9.81 \text{ m s}^{-2}} = 0.3 \text{ m} + \frac{p_2}{\gamma_w} + \frac{(1.7 \text{ m s}^{-1})^2}{2 \text{ x} 9.81 \text{ m s}^{-2}}$$

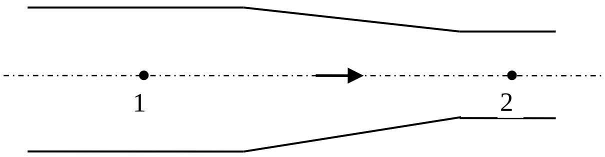
$$\Rightarrow \frac{p_2}{\gamma_w} = \frac{9.3 \text{ x} 10^3 \text{ N m}^{-2}}{9810 \text{ N m}^{-3}} + \frac{(3.1 \text{ m s}^{-1})^2}{2 \text{ x } 9.81 \text{ m s}^{-2}} - 0.3 \text{ m} - \frac{(1.7 \text{ m s}^{-1})^2}{2 \text{ x } 9.81 \text{ m s}^{-2}}$$

$$\Rightarrow \frac{p_2}{\gamma_w} = 0.991 \; m \text{ of water}$$

$\Rightarrow p_2 = 0.991 \text{ x } \gamma_w = 0.991 \text{ x } 9810 \; N \, m^{-3} = 9717 \; N \, m^{-2} = 9.717 \text{ x } 10^3 \; N \, m^{-2}$

Example. Water flows through a pipeline which reduces in cross-section. The centreline of the pipe is horizontal. If $V_1 = 1.54 \; m \, s^{-1}$ and $V_2 = 2.65 \; m \, s^{-1}$, $p_1 = 20 \text{ x } 10^3 \; N \, m^{-2}$ and $p_2 = 16.89 \text{ x } 10^3 \; N \, m^{-2}$, what is the energy loss between sections 1 and 2? Give the answer in metres of water.

Solution.



Assuming the datum to pass through the centreline of the pipe that is horizontal, we have datum head at point 1, Z_1, and datum head at point 2, Z_2, equal to zero.

Applying Bernoulli's equation between points 1 and 2, we have,

$$Z_1 + \frac{p_1}{\gamma_w} + \frac{V_1^2}{2g} = Z_2 + \frac{p_2}{\gamma_w} + \frac{V_2^2}{2g} + h_L$$

$$\Rightarrow$$

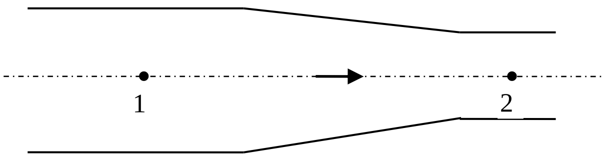
$$0 + \frac{20 \text{ x} 10^3 \text{ N m}^{-2}}{9810 \text{ N m}^{-3}} + \frac{(1.54 \text{ m s}^{-1})^2}{2 \text{ x } 9.81 \text{ m s}^{-2}} = 0 + \frac{16.89 \text{ x} 10^3 \text{ N m}^{-2}}{9810 \text{ N m}^{-3}} + \frac{(2.65 \text{ m s}^{-1})^2}{2 \text{ x } 9.81 \text{ m s}^{-2}} + h_L$$

$$\Rightarrow h_L = \frac{20 \text{ x} 10^3 \text{ N m}^{-2}}{9810 \text{ N m}^{-3}} + \frac{(1.54 \text{ m s}^{-1})^2}{2 \text{ x } 9.81 \text{ m s}^{-2}} - \frac{16.89 \text{ x} 10^3 \text{ N m}^{-2}}{9810 \text{ N m}^{-3}} - \frac{(2.65 \text{ m s}^{-1})^2}{2 \text{ x } 9.81 \text{ m s}^{-2}}$$

$\Rightarrow h_L = 0.080 \; m$ of water

Example. Water flows through a straight pipeline that reduces in diameter from section 1 to 2. The centreline of the pipe is horizontal. If $V_1 = 1.54 \; m \, s^{-1}$, $p_1 = 20 \text{ x } 10^3 \; N \, m^{-2}$ and $V_2 = 2.65 \; m \, s^{-1}$,what is p_2 in $N \, m^{-2}$ and as the equivalent head of water? Assume no loss of energy.

Solution.



Assuming the datum to pass through the centreline of the pipe that is horizontal, we have datum head at point 1, Z_1, and datum head at point 2, Z_2, equal to zero.

Applying Bernoulli's equation between points 1 and 2, we have,

$$Z_1 + \frac{p_1}{\gamma_w} + \frac{V_1^2}{2g} = Z_2 + \frac{p_2}{\gamma_w} + \frac{V_2^2}{2g}$$

$$\Rightarrow\ 0 + \frac{20 \text{ x } 10^3 \text{ N m}^{-2}}{9810 \text{ N m}^{-3}} + \frac{(1.54 \text{ m s}^{-1})^2}{2 \text{ x } 9.81 \text{ m s}^{-2}} = 0 + \frac{p_2}{\gamma_w} + \frac{(2.65 \text{ m s}^{-1})^2}{2 \text{ x } 9.81 \text{ m s}^{-2}}$$

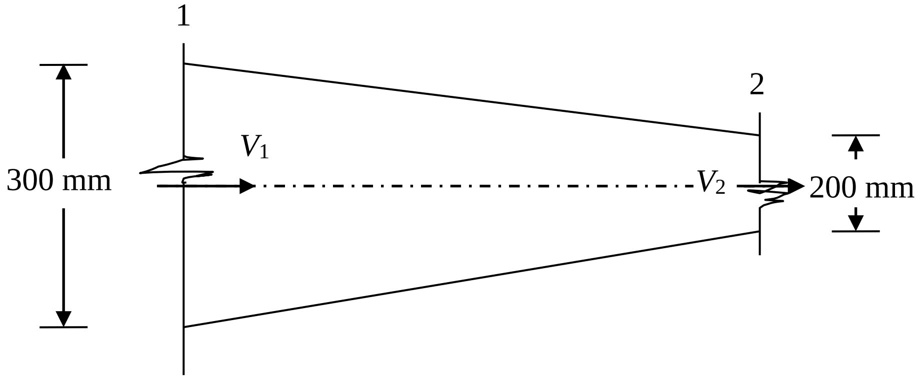
$$\Rightarrow\ \frac{p_2}{\gamma_w} = \frac{20 \text{ x } 10^3 \text{ N m}^{-2}}{9810 \text{ N m}^{-3}} + \frac{(1.54 \text{ m s}^{-1})^2}{2 \text{ x } 9.81 \text{ m s}^{-2}} - \frac{(2.65 \text{ m s}^{-1})^2}{2 \text{ x } 9.81 \text{ m s}^{-2}} = 1.802\ m \text{ of water}$$

$$\Rightarrow p_2 = 1.802\ m \text{ x } 9810\ N\ m^{-3} = 17675\ N\ m^{-2} = 17.675 \text{ x } 10^3\ N\ m^{-2}$$

Example. A tapering pipe has diameters 300 mm and 200 mm at sections 1 and 2. The velocity of flow at section 1 is 3 m s^{-1}. Determine the velocity head at sections 1 and 2 and the discharge through the pipe.

Solution.

For convenience, let us assume the tapering pipe to be laid horizontal so that the datum can be assumed to pass through the centreline of the pipe. This makes the datum head at sections 1 and 2 to be zero, that is, $Z_1 = Z_2 = 0$.



Area of cross-section at section 1 of pipe, $A_1 = \left(\frac{\pi}{4} d_1^2\right) = \left(\frac{\pi}{4}\, x\, 0.3^2\right) = 0.0707\ m^2$

Area of cross-section at section 2 of pipe, $A_2 = \left(\frac{\pi}{4} d_2^2\right) = \left(\frac{\pi}{4}\, x\, 0.2^2\right) = 0.031429\ m^2$

Rate of flow, $Q = A_1V_1 = 0.070714\ m^2 \text{ x } 3\ m\ s^{-1} = 0.212143\ m^3\ s^{-1}$

As per continuity principle,

$Q = A_1V_1 = A_2V_2$

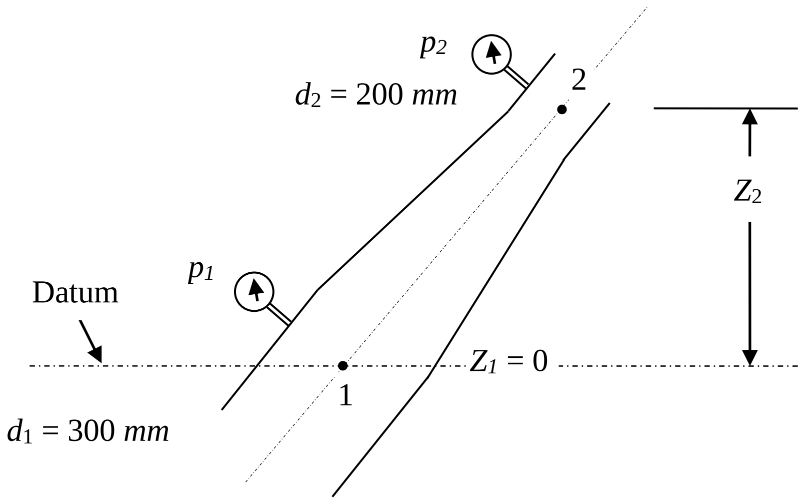
$\Rightarrow V_2 = \frac{Q}{A_2} = \frac{0.212143}{0.031429} = 6.75\ m\ s^{-1}$

Velocity head at section 1 = $\frac{V_1^2}{2g} = \frac{3^2}{2 \text{ x } 9.81}$ = 0.459 m

Velocity head at section 2 = $\frac{V_2^2}{2g} = \frac{6.75^2}{2 \text{ x } 9.81}$ = 2.322 m

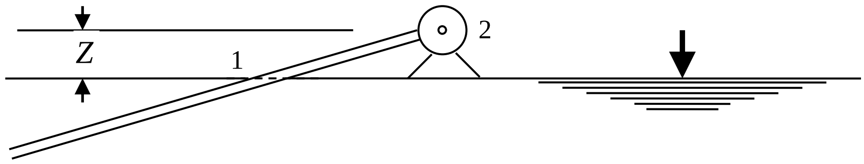
Example. An inclined pipeline carrying water has a diameter of 300 mm at the lower end and the pressure is 195.2 kN m^{-2}. The diameter at the upper end is 200 mm and the pressure is 147.15 kN m^{-2}. Water is flowing at the rate of 50 litres per second. Find the difference in datum heads.

Solution.



Example. The suction pipe of a pump rises at a slope of 1 vertical in 5 along the pipe and water passes through it at 1.8 $m\ s^{-1}$. If dissolved air is released when pressure falls to more than 70 $kN\ m^{-2}$ below atmospheric pressure, find the greatest practicable length of pipe neglecting friction. Assume that the water in the sump is at rest.

Solution.



1 Intersection of free surface of water and suction pipe

2 Centre of pump

Z Vertical height of centre of pump from free water surface (or) suction height

L Greatest practicable length of suction pipe (measured from point 1 to point 2)

Slope of suction pipe = 1 vertical : 5 along the pipe length

Let us assume the datum to pass through the free surface of water. Hence, datum head at point 1, $Z_1 = 0$

As water starts from rest at the free surface, the velocity of flow at point 1, V_1 = 0

Pressure at free surface of water is atmospheric. Hence, the pressure, p_1, in excess of atmospheric pressure at the free surface is zero. So, the pressure head at point 1 as $\frac{p_1}{\gamma_w} = 0$.

Datum head at point 2, $Z_2 = Z = L / 5$

Velocity of flow through the suction pipe, $V_2 = 1.8\ m\ s^{-1}$

Hence, kinetic head at point 2 = $\frac{V_2^2}{2g} = \frac{1.8^2}{2 \times 9.81} = 0.165\ m$ of water

In order to avoid separation or to prevent the release of dissolved air in the suction pipe, the pressure at the centre of the pump should not fall to more than 70 $kN\ m^{-2}$ below atmospheric pressure. That is, $p_2 = -70\ kN\ m^{-2}$

$= -70 \times 10^3\ N\ m^{-2}$

Hence, pressure head at point 2 = $\frac{p_2}{\gamma_w} = -\frac{70 \times 10^3}{9810} = -7.136\ m$ of water

Now, applying Bernoulli's equation between points '1' and '2', we have,

$$Z_1 + \frac{p_1}{\gamma_w} + \frac{V_1^2}{2g} = Z_2 + \frac{p_2}{\gamma_w} + \frac{V_2^2}{2g}$$

(Note: in applying the Bernoulli's equation, the pipe friction has been neglected).

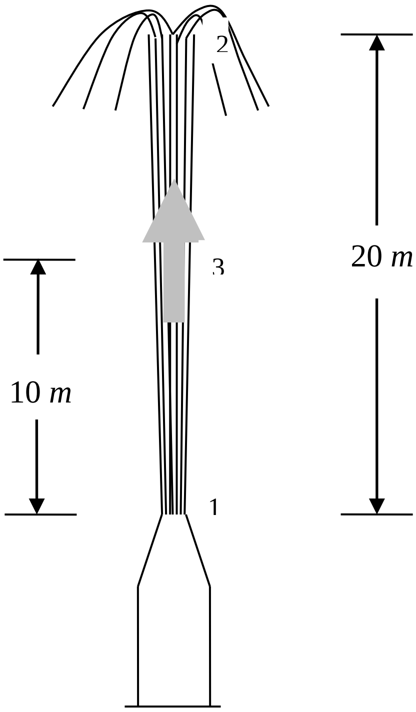
$\Rightarrow 0 + 0 + 0 = \frac{L}{5} - 7.136 + 0.165$

$\Rightarrow \frac{L}{5} = 7.136 - 0.165 = 6.971$

$\Rightarrow L = 5 \times 6.971 = 34.855\ m$

Example. A jet of water is initially 120 *mm* in diameter and when directed vertically upwards reaches a maximum height of 20 *m*. Assuming that the jet remains circular determine the rate of water flowing and the diameter of the jet at a height of 10 *m*.

Solution.



Applying Bernoulli's equation between the points 1 and 2,

$$Z_1 + \frac{p_1}{\gamma_w} + \frac{V_1^2}{2g} = Z_2 + \frac{p_2}{\gamma_w} + \frac{V_2^2}{2g}$$

As pressure at points 1 and 2 are atmospheric,

$$Z_1 + \frac{p_{atm}}{\gamma_w} + \frac{V_1^2}{2g} = Z_2 + \frac{p_{atm}}{\gamma_w} + \frac{V_2^2}{2g}$$

$$\Rightarrow \; Z_1 + \frac{V_1^2}{2g} = Z_2 + \frac{V_2^2}{2g}$$

$$\Rightarrow 0 + \frac{V_1^2}{2g} = 20 + \text{zero kinetic head}$$ (since velocity of flow of jet at the maximum height of 20 m is zero)

$$\Rightarrow \frac{V_1^2}{2g} = 20\,\text{m}$$

$$\Rightarrow V_1 = 19.81 \; m \; s^{-1}$$

Rate of flow, $Q = a_1 V_1 = \left(\frac{\pi}{4} d_1^2\right) V_1 = \left(\frac{\pi}{4} \text{ x } 0.12^2\right) \text{x } 19.81 = 0.224 \; m^3 \; s^{-1}$

Applying Bernoulli's equation between the points 1 and 3,

$$Z_1 + \frac{p_1}{\gamma_w} + \frac{V_1^2}{2g} = Z_3 + \frac{p_3}{\gamma_w} + \frac{V_3^2}{2g}$$

As pressure at points 1 and 3 are atmospheric

$$Z_1 + \frac{p_{atm}}{\gamma_w} + \frac{V_1^2}{2g} = Z_3 + \frac{p_{atm}}{\gamma_w} + \frac{V_3^2}{2g}$$

$$\Rightarrow\ Z_1 + \frac{V_1^2}{2g} = Z_3 + \frac{V_3^2}{2g}$$

$$\Rightarrow 0 + 20 = 10 + \frac{V_3^2}{2g}$$

$$\Rightarrow\ \frac{V_3^2}{2g} = 20 - 10 = 10 \text{ m}$$

$$\Rightarrow \text{V}_3 = 14 \text{ m s}^{-1}$$

As per continuity principle,

$Q = a_1V_1 = a_3V_3$

where a_1 and a_3 are the cross-sectional areas of jet at the exit of nozzle and at a height of 10 m above the exit of nozzle, respectively. The symbols V_1 and V_3 denote the velocity of jet at points 1 and 3 of the jet respectively.

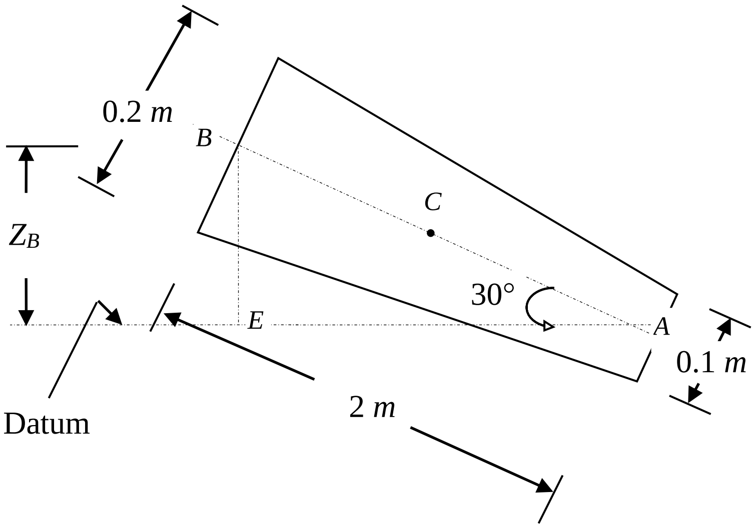
$$Q = \left(\frac{\pi}{4} d_1^2\right)V_1 = \left(\frac{\pi}{4} d_3^2\right)V_3$$

$$\Rightarrow 0.224\ m^3\ s^{-1} = \left(\frac{\pi}{4} \text{ x } d_1^2\right) \text{x}\, 19.81 = \left(\frac{\pi}{4} \text{ x } d_2^2\right) \text{x}\, 14$$

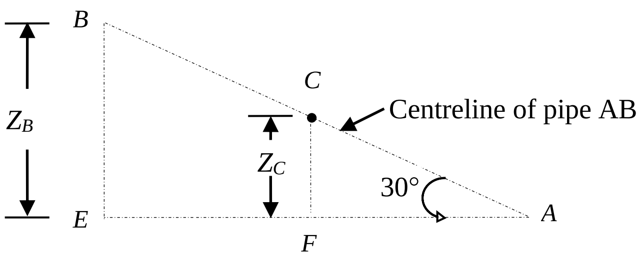
$$\Rightarrow d_2 = 0.143\ m$$

Example. A pipe AB tapering uniformly from a diameter of 0.1 m at A to 0.2 m at B over a length of 2 m carries water. Pressures at A and B are respectively 2.0 and 2.3 bar. The centreline of the pipe slopes upwards from A to B at an angle of 30°. Determine the flow through the pipe and the pressure at C, the mid-point of pipe AB, ignoring the losses.

Solution.



Diameter of pipe at A, $D_A = 0.1\ m$
Diameter of pipe at B, $D_B = 0.2\ m$
Length of pipe AB, $L = 2\ m$
Inclination of pipe AB with horizontal, $\theta = 30°$
Pressure at A, $p_A = 2.0\ bar = 2 \times 10^5\ N\ m^{-2}$
Pressure at B, $p_B = 2.3\ bar = 2.3 \times 10^5\ N\ m^{-2}$
Assuming the datum to pass through A, datum head at A, $Z_A = 0$



From right=angled triangle AEB,

$$\sin 30^\circ = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{EB}{AB} = \frac{Z_B}{2}$$

$$\Rightarrow Z_B = 2 \sin 30^\circ = 2 \times 0.5 = 1.0\ \text{m}$$

Datum head at B, $Z_B = 1.0\ m$

Applying equation of continuity between A and B,

$$Q = A_A V_A = A_B V_B$$

$$\Rightarrow \left(\frac{\pi}{4} D_A^2\right) V_A = \left(\frac{\pi}{4} D_B^2\right) V_B$$

$$\Rightarrow D_A^2 V_A = D_B^2 V_B$$

$$\Rightarrow (0.1)^2 V_A = (0.2)^2 V_B$$

$$\Rightarrow V_A = \frac{(0.2)^2}{(0.1)^2} V_B = 4V_B$$

Assuming the flow to occur from B to A as the pressure at B is higher than A and applying the Bernoulli's equation between A and B, ignoring the losses between B and A, we have,

$$Z_B + \frac{p_B}{\gamma} + \frac{V_B^2}{2g} = Z_A + \frac{p_A}{\gamma} + \frac{V_A^2}{2g}$$

Substituting the values of Z_B, Z_A, p_B and p_A and putting $V_A = 4V_B$, we have,

$$1 + \frac{2.3 \text{ x } 10^5}{9810} + \frac{V_B^2}{2g} = 0 + \frac{2.0 \text{ x } 10^5}{9810} + \frac{(4V_B)^2}{2g}$$

$$\Rightarrow \frac{(4V_B)^2}{2g} - \frac{V_B^2}{2g} = \frac{2.3 \text{ x } 10^5}{9810} - \frac{2.0 \text{ x } 10^5}{9810} + 1$$

$$\Rightarrow \frac{15V_B^2}{2g} = \frac{0.3 \text{ x } 10^5}{9810} + 1$$

$\Rightarrow V_B = 2.304 \text{ m s}^{-1}$

Hence, $V_A = 4V_B = 4 \text{ x } 2.304 = 9.216 \text{ m s}^{-1}$

$Q = A_A V_A = \left(\frac{\pi}{4} D_A^2\right) V_A = \left(\frac{\pi}{4} \text{ x } 0.1^2\right) \text{ x } 9.216 =$ **$0.0724 \text{ m}^3 \text{ s}^{-1}$**

As C is mid-way between A and B, diameter of pipe at C,

$$D_C = \frac{D_B + D_A}{2} = \frac{0.2 + 0.1}{2} = 0.15\, m$$

Length of C from A = $\frac{\overline{AB}}{2} = \frac{2}{2} = 1 \ m$

From right-angled triangle AFC, $\sin 30^\circ = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{FC}{AC} = \frac{Z_C}{1}$

$$\Rightarrow Z_C = 1 \sin 30^\circ = 1 \text{ x } 0.5 = 0.5 \text{ m}$$

Applying equation of continuity between A and C,

$$Q = A_A V_A = A_C V_C$$

$$\Rightarrow \left(\frac{\pi}{4} D_A^2\right) V_A = \left(\frac{\pi}{4} D_C^2\right) V_C$$

$$\Rightarrow D_A^2 V_A = D_C^2 V_C$$

$$\Rightarrow (0.1)^2 \text{ x } 9.216 = (0.15)^2 V_C$$

$$\Rightarrow V_C = \frac{(0.1)^2}{(0.15)^2} \text{ x } 9.216 = 4.096 \text{ m s}^{-1}$$

Applying the Bernoulli's equation between C and A, ignoring the losses between C and A, we have,

$$Z_C + \frac{p_C}{\gamma} + \frac{V_C^2}{2g} = Z_A + \frac{p_A}{\gamma} + \frac{V_A^2}{2g}$$

Substituting the values of Z_C, Z_A, p_A, V_C and V_A, we have,

$$0.5 + \frac{p_C}{9810} + \frac{4.096^2}{2g} = 0 + \frac{2.0 \text{ x } 10^5}{9810} + \frac{9.216^2}{2g}$$

$$\Rightarrow \frac{p_C}{9810} = 23.361 \text{ m of water}$$

$\Rightarrow p_C = 229174 \text{ N m}^{-2}$

$\Rightarrow$ **$p_C = 2.29174 \text{ x } 10^5 \text{ N m}^{-2} = 2.29174$ bar**

Example. Air enters a compressor at the rate of 0.5 kg s^{-1} with a velocity of 6.4 m s^{-1}. The specific volume and pressure of air at entry to compressor are respectively 0.85 m^3 kg^{-1} and 1 bar. Air leaves the compressor at a pressure of 6.9 bar with a specific volume of 0.16 m^3 kg^{-1}. The velocity of air leaving the compressor is 4.7 m s^{-1}. The internal energy of air at exit is greater than that at entry by 85 kJ kg^{-1}. The compressor is fitted with a cooling system which removes heat at the rate of 60 kJ s^{-1}. Calculate the power required to drive the compressor and the cross-sectional areas of the inlet and outlet pipes.

Solution.

Let the entry to and exit of compressor be denoted by the subscripts '1' and '2' respectively.

Mass of air entering the compressor per unit time, $\rho_1 Q_1 = 0.5 \; kg \; s^{-1}$

Velocity of air at entry to compressor, $V_1 = 6.4 \; m \; s^{-1}$

Specific volume of air at entry to compressor, $\left(\frac{1}{\rho_1}\right) = 0.85 \; m^3 \; kg^{-1}$

Hence volume flow rate of air at entry to compressor, $Q_1 = (\rho_1 Q_1)\left(\frac{1}{\rho_1}\right)$

$= 0.5 \text{ x } 0.85$

$= 0.425 \; m^3 \; s^{-1}$

Mass density of air at entry to compressor, $\rho_1 = \frac{1}{\text{Specific volume}} = \frac{1}{0.85}$

$= = 1.176 \text{ kg m}^{-3}$

Pressure of air at entry to compressor, $p_1 = 1 \; bar = 1 \text{ x } 10^5 \text{ N m}^{-2}$

Pressure of air at exit of compressor, $p_2 = 6.9\ bar = 6.9 \times 10^5$ N m^{-2}

Specific volume of air at exit of compressor, $\left(\frac{1}{\rho_2}\right) = 0.16\ m^3\ kg^{-1}$

Mass density of air at exit of compressor, $\rho_2 = \frac{1}{\text{Specific volume}} = \frac{1}{0.16}$

$= 6.25\,\text{kg m}^{-3}$

Velocity of air at exit of compressor, $V_2 = 4.7\ m\ s^{-1}$

Applying equation of continuity between entry and exit of compressor,

$\rho_1 Q_1 = \rho_2 Q_2$

$\Rightarrow 1.176 \times 0.425 = 6.25Q_2$

$\Rightarrow Q_2 = \frac{1.176 \times 0.425}{6.25} = 0.08\ \text{m}^3\ \text{s}^{-1}$

$Q_1 = A_1V_1$

$\Rightarrow 0.425 = A_1 \times 6.4$

$\Rightarrow$ **A_1** $= \frac{0.425}{6.4} =$ **$0.0664\ \text{m}^2$**

Volume flow rate of air at exit of compressor, $Q_2 = A_2V_2$

$\Rightarrow 0.08 = A_2 \times 4.7$

$\Rightarrow$ **A_2** $= \frac{0.08}{4.7} =$ **$0.017\ \text{m}^2$**

Solution incomplete

KINETIC ENERGY CORRECTION FACTOR

The derivation of the Bernoulli's equation applicable for steady one-dimensional flow of a fluid has been carried out for a streamtube assuming uniform velocity across the inlet and outlet sections. But, in case of a real fluid flowing in a pipe or over a solid surface, the velocity of flow will be zero at the wall of the pipe or the solid boundary and will increase with distance from the wall of the pipe up to the centre of the pipe or from the boundary of the solid surface. As the velocity of flow increases with the distance from the solid boundary, the kinetic energy per unit weight of the fluid will also increase in a similar manner. If the cross-section of the flow is assumed to be composed of a series of small elements of area δA and the velocity of flow normal (perpendicular) to each element is v, the total kinetic energy passing through the entire cross-section in unit time can be computed by determining the kinetic energy passing through the element in unit time and then summing up by integrating over the whole area of the cross-section.

Mass of fluid flowing through element of area δA, forming part of the whole cross-section of flow of area A, in unit time
= (mass density of fluid) x (cross-sectional area of element) x
(velocity of flow normal to the element)
$= \rho.\delta A.v$

Kinetic energy per unit time passing through the element
$= \frac{1}{2}$ x mass of fluid flowing per unit time through the element x
(velocity of flow normal to the element)^2
$= \frac{1}{2}(\rho.\delta A.v)v^2 = \frac{1}{2}\rho.\delta A.v^3$

Total kinetic energy passing per unit time through the whole cross-section
$= \int \frac{1}{2}\rho v^3 \delta A$

Weight of fluid passing in unit time through the element of area δA
= (specific weight of fluid) x (volume rate of flow through the element)
$= \gamma \text{ x } (v.\delta A) = \rho g v.\delta A$

Total weight of fluid passing in unit time through the entire cross-section
$= \int \rho g v.\delta A$

Hence, taking into account the variation of velocity across the stream,

True kinetic energy per unit weight of flowing fluid

$$= \frac{\text{Total kinetic energy passing per unit time through the whole crosss - section}}{\text{Total weight of fluid passing per unit time through the whole cross - section}}$$

$$= \frac{\int \frac{1}{2}\rho v^3 \delta A}{\int \rho g v.\delta A} = \frac{\frac{1}{2}\int \rho v^3 \delta A}{\int \rho g v \delta A} \qquad (1)$$

The above equation is not the same as $\frac{V^2}{2g}$, where V is the mean velocity of flow normal to the whole cross-section of flow passage.

Mean velocity of flow, $V = \frac{\int v.\delta A}{A}$ (2)

Thus, true kinetic energy per unit weight of fluid = $\alpha\left(\frac{V^2}{2g}\right)$ (3)

where α is the kinetic energy correction factor. The value of α is dependent upon the shape of the cross-section and the velocity distribution. For a circular pipe, assuming Prandtl's one-seventh power law, $v = v_{\max}\left(\frac{y}{R}\right)^{1/7}$, where v is the velocity of flow at a distance y from the wall of the pipe of radius R, the value of kinetic energy correction factor α = 1.058.

Example. A liquid flows through a circular pipe 0.6 m diameter. Measurements of velocity taken at intervals along a diameter are:

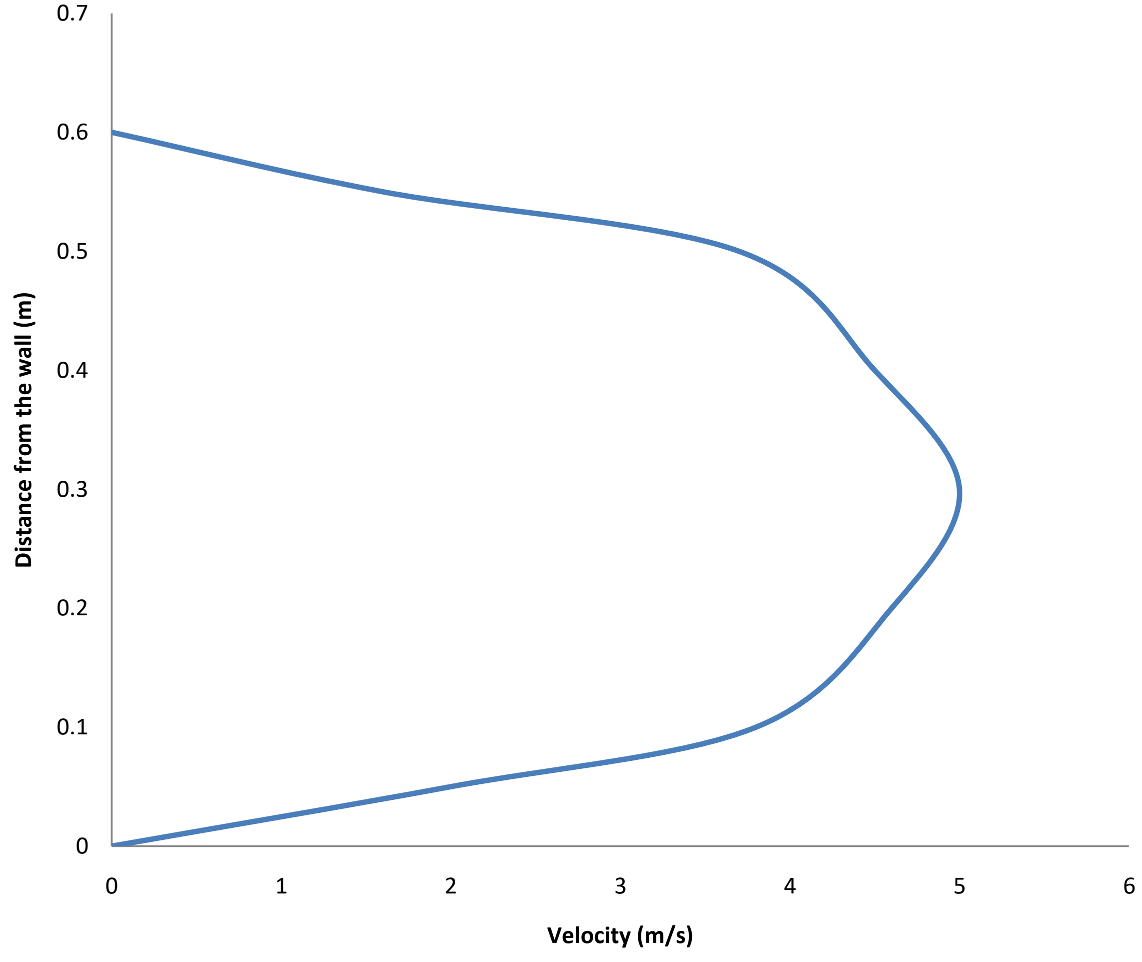
Distance from the wall of the pipe (m)	0	0.5	0.1	0.2	0.3	0.4
Velocity of flow (m s^{-1})	0	2.0	3.8	4.6	5.0	4.5

Distance from the wall of the pipe (m)	0.5	0.55	0.6
Velocity of flow (m s^{-1})	3.7	1.6	0.0

Draw the velocity profile, compute the mean velocity and also determine the kinetic energy correction factor.

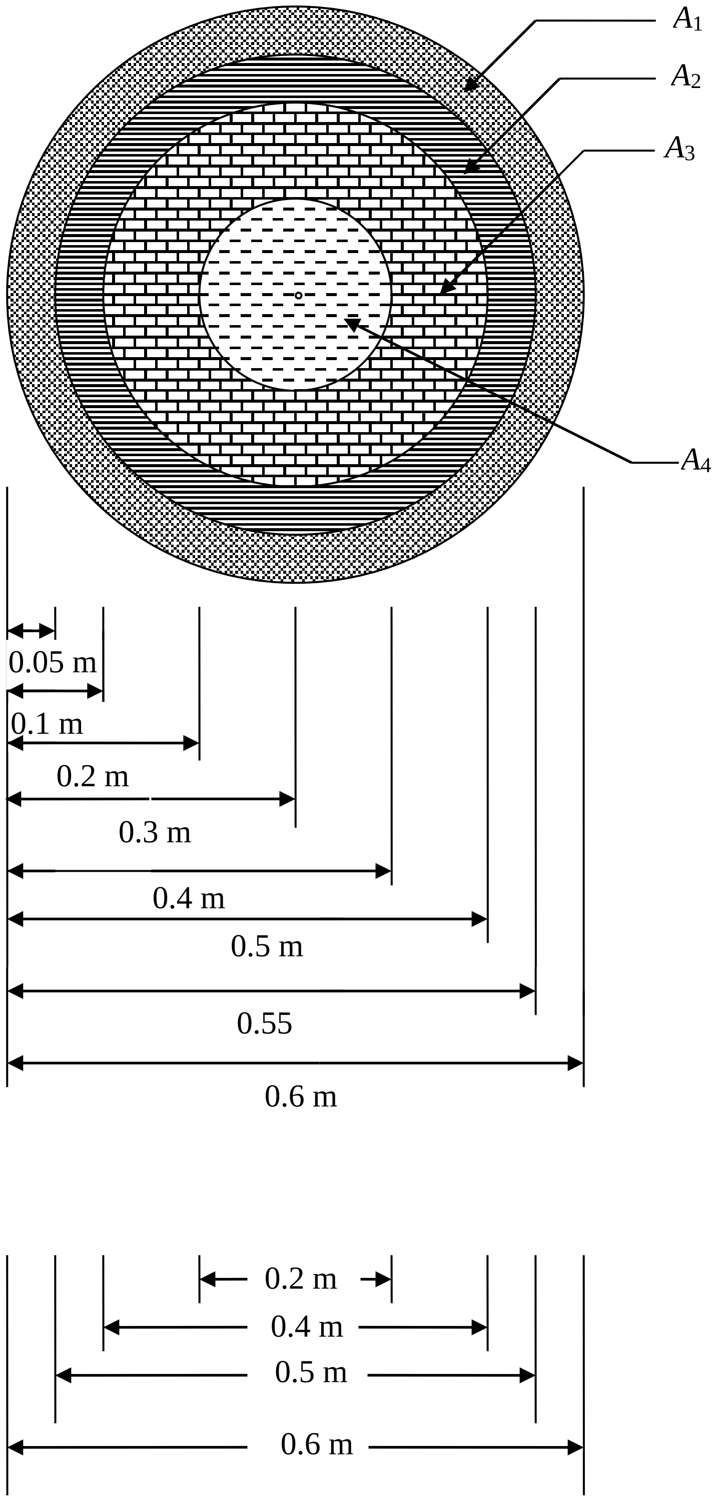
Solution.

Figure below shows the velocity profile in the pipe section.



Velocity Profile

Figure below shows the cross-section of pipe of diameter 0.6 m with the elemental areas A_1, A_2, A_3 and A_4 configured depending upon the velocity of flow provided at different distances from the wall of the pipe (as stated in the statement of the problem).



Kinetic energy per unit time passing through area A_1 of the pipe

$$= \frac{1}{2}(\rho A_1 V_1)V_1^2 = \frac{1}{2}\rho A_1 V_1^3$$

V_1 = mean velocity in area A_1

$$= \frac{\left(\frac{V_{0\,\mathrm{m}} + V_{0.05\,\mathrm{m}}}{2}\right) + \left(\frac{V_{0.6\,\mathrm{m}} + V_{0.55\,\mathrm{m}}}{2}\right)}{2}$$

$$= \frac{\left(\frac{0+2}{2}\right) + \left(\frac{0+1.6}{2}\right)}{2} = \frac{1+0.8}{2} = 0.9\, m\ s^{-1}$$

Area $A_1 = \frac{\pi}{4}\left\{(0.6-0)^2 - (0.55-0.05)^2\right\} = \frac{\pi}{4}\left\{0.6^2 - 0.5^2\right\} = 0.086429\ m^2$

Hence, kinetic energy per unit time passing through area A_1 of the pipe

$$= \frac{1}{2}\rho \text{ x } 0.086429 \text{ x } 0.9^3 = 0.031503\rho$$

Kinetic energy per unit time passing through area A_2 of the pipe

$$= \frac{1}{2}(\rho A_2 V_2)V_2^2 = \frac{1}{2}\rho A_2 V_2^3$$

V_2 = mean velocity in area A_2

$$= \frac{\left(\frac{V_{0.05\,\mathrm{m}} + V_{0.1\,\mathrm{m}}}{2}\right) + \left(\frac{V_{0.55\,\mathrm{m}} + V_{0.5\,\mathrm{m}}}{2}\right)}{2}$$

$$= \frac{\left(\frac{2+3.8}{2}\right) + \left(\frac{1.6+3.7}{2}\right)}{2} = \frac{2.9+2.65}{2} = 2.775\, m\ s^{-1}$$

Area $A_2 = \frac{\pi}{4}\left\{(0.55-0.05)^2 - (0.5-0.1)^2\right\} = \frac{\pi}{4}\left\{0.5^2 - 0.4^2\right\} = 0.0070714\ m^2$

Hence, kinetic energy per unit time passing through area A_2 of the pipe

$$= \frac{1}{2}\rho \text{ x } 0.070714 \text{ x } 2.775^3 = 0.755552\rho$$

Kinetic energy per unit time passing through area A_3 of the pipe

$$= \frac{1}{2}(\rho A_3 V_3)V_3^2 = \frac{1}{2}\rho A_3 V_3^3$$

V_3 = mean velocity in area A_3

$$= \frac{\left(\frac{V_{0.1\,\mathrm{m}} + V_{0.2\,\mathrm{m}}}{2}\right) + \left(\frac{V_{0.5\,\mathrm{m}} + V_{0.4\,\mathrm{m}}}{2}\right)}{2}$$

$$= \frac{\left(\frac{3.8+4.6}{2}\right) + \left(\frac{3.7+4.5}{2}\right)}{2} = \frac{4.2+4.1}{2} = 4.15\, m\ s^{-1}$$

Area $A_3 = \frac{\pi}{4}\left\{(0.5-0.1)^2 - (0.4-0.2)^2\right\} = \frac{\pi}{4}\left\{0.4^2 - 0.2^2\right\} = 0.094286\ m^2$

Hence, kinetic energy per unit time passing through area A_3 of the pipe

$$= \frac{1}{2}\rho \text{ x } 0.094286 \text{ x } 4.15^3 = 3.369469\rho$$

Kinetic energy per unit time passing through area A_4 of the pipe

$$= \frac{1}{2}(\rho A_4 V_4)V_4^2 = \frac{1}{2}\rho A_4 V_4^3$$

V_4 = mean velocity in area A_4

$$= \frac{\left(\frac{V_{0.2\,m} + V_{0.3\,m}}{2}\right) + \left(\frac{V_{0.4\,m} + V_{0.3\,m}}{2}\right)}{2}$$

$$= \frac{\left(\frac{4.6 + 5.0}{2}\right) + \left(\frac{4.5 + 5.0}{2}\right)}{2} = \frac{4.8 + 4.75}{2} = 4.775\, m\ s^{-1}$$

Area $A_4 = \frac{\pi}{4}\left\{(0.4 - 0.2)^2\right\} = \frac{\pi}{4}\left\{0.2^2\right\} = 0.031429\ m^2$

Hence, kinetic energy per unit time passing through area A_4 of the pipe

$$= \frac{1}{2}\rho \text{ x } 0.031429 \text{ x } 4.775^3 = 1.710885\rho$$

Total kinetic energy passing in unit time through the entire pipe of area A

$= 0.031503\rho + 0.755552\rho + 3.369469\rho + 1.710885\rho = 5.867\rho$

Weight of fluid passing in unit time through area A_1 of the pipe

$= \rho g V_1 A_1$

$= \rho$ x 9.81 x 0.9 x 0.086429 = 0.763082ρ

Weight of fluid passing in unit time through area A_2 of the pipe

$= \rho g V_2 A_2$

$= \rho$ x 9.81 x 2.775 x 0.070714 = 1.92503ρ

Weight of fluid passing in unit time through area A_3 of the pipe

$= \rho g V_3 A_3$

$= \rho$ x 9.81 x 4.15 x 0.094286 = 3.838524ρ

Weight of fluid passing in unit time through area A_4 of the pipe

$= \rho g V_4 A_4$

$= \rho$ x 9.81 x 4.775 x 0.031429 = 1.472221ρ

Total weight of fluid passing in unit time through the entire pipe of area A

$= 0.763082\rho + 1.92503\rho + 3.838524\rho + 1.472221\rho$

$= 8\rho$

True kinetic energy per unit weight = $\dfrac{\text{Total kinetic energy passing in unit time}}{\text{Total weight passing in unit time}}$

$$= \frac{5.867\rho}{8\rho} = 0.733375$$

Mean velocity in the pipe of area A, $V = \dfrac{A_1V_1 + A_2V_2 + A_3V_3 + A_4V_4}{A}$

$$\Rightarrow V = \frac{(0.086429 \text{ x } 0.9) + (0.070714 \text{ x } 2.775) + (0.094286 \text{ x } 4.15) + (0.031429 \text{ x } 4.775)}{\left(\frac{\pi}{4} \text{ x } 0.6^2\right)}$$

$= 2.883\ m\ s^{-1}$

True kinetic energy per unit weight = $\alpha \dfrac{V^2}{2g}$

$$\Rightarrow 0.733 = \alpha\left(\frac{2.883^2}{2 \text{ x } 9.81}\right)$$

$$\Rightarrow \alpha = \frac{0.733}{\left(\frac{2.883^2}{2 \text{ x } 9.81}\right)} = 1.730$$

POWER DEVELOPED BY A STREAM OF FLUID

A stream of fluid possesses pressure energy due to its pressure p, kinetic energy due to its velocity V and potential energy due to its elevation Z. The total energy possessed by the stream of fluid is the sum of pressure energy, kinetic energy and potential energy and the total energy per unit weight of flowing fluid is given by

$$H = Z + \frac{p}{\gamma} + \frac{V^2}{2g}$$

If the weight of fluid flowing per unit time is known, the power developed by the stream can be calculated as follows:

Power developed = Energy per unit time
= (Weight per unit time) x (Energy per unit weight)

If Q is the volume rate of flow, then weight of fluid flowing per unit time

= specific weight of fluid x volume rate of flow = γQ

Hence, power developed = $(\gamma Q)H = (\gamma Q)\left(Z + \frac{p}{\gamma} + \frac{V^2}{2g}\right)$ (4)

Example. Water is drawn from a reservoir through a pipeline at a rate of 0.13 $m^3\ s^{-1}$. The water level in the reservoir is 240 m above datum. The outlet of pipeline is at the datum level and is fitted with a nozzle to produce a high speed jet to drive a Pelton turbine. If the velocity of the jet issued from the nozzle fitted at the outlet of the pipeline is 66 $m\ s^{-1}$, compute the (a) the power of the water jet issued from the nozzle (b) the water power supplied from the reservoir (c) the head required to overcome losses (d) the efficiency of the pipeline and the nozzle.

Solution.

Volume rate of flow through the pipeline (or nozzle), $Q = 0.13\ m^3\ s^{-1}$
Velocity of jet issued from the nozzle, V = 66 $m\ s^{-1}$
As the nozzle will be discharging water jet into the atmosphere, pressure energy at the tip of nozzle, $p = 0$; hence, pressure head, $\frac{p}{\gamma} = 0$

As the nozzle is fitted at the outlet of the pipeline located at datum level, potential head, $Z = 0$

(a) Power of jet issued from the nozzle $= (\gamma Q)\left(Z + \frac{p}{\gamma} + \frac{V^2}{2g}\right)$

$$= (\gamma Q)\left(0 + 0 + \frac{V^2}{2g}\right)$$

$$= 9810 \text{ x } 0.13 \text{ x } \left(\frac{66^2}{2 \text{ x } 9.81}\right)$$

$$= 283140\ W = 283.14\ kW$$

(b) At the reservoir surface, the pressure is atmospheric. Hence, the pressure, p, in excess of atmospheric pressure, is equal to zero; hence, pressure head, $\frac{p}{\gamma} = 0$

As water in the reservoir is in static condition, the velocity of water at the free surface, V = 0; hence, kinetic head, $\frac{V^2}{2g} = 0$

As the water surface in the reservoir is at an elevation of 240 m from the datum, the potential head, Z = 240 m

Hence, water power supplied from the reservoir = $(\gamma Q)\left(Z + \frac{p}{\gamma} + \frac{V^2}{2g}\right)$

$= (\gamma Q)(240 + 0 + 0)$

$= 9810 \text{ x } 0.13 \text{ x } 240$

$= 306072\ W = 306.072\ kW$

(c) Let H_1 = Total head at the reservoir (inlet to pipe) = 240 m

Let H_2 = Total head at the nozzle exit

Let h_L be the head lost in transmission of water from reservoir to the nozzle exit

From (b), Power supplied from the reservoir = 306.072 kW
From (a), Power of jet issued from the nozzle = 283.14 kW

Hence, power lost in transmission = 306.072 – 283.14 = 22.932 kW

$= \gamma Q h_L$

$\Rightarrow 22932 = 9810 \text{ x } 0.13 \text{ x } h_L$

$\Rightarrow h_L = \dfrac{22932}{9810 \text{ x } 0.13} = 17.982\ m$

(d) Efficiency of transmission = $\dfrac{\text{Power of jet supplied by the nozzle}}{\text{Power supplied from the reservoir}}$

$= \dfrac{283.14}{306.072} = 0.925 \text{ (or) } 92.5\%$

Example. A pump discharges 2 $m^3\ s^{-1}$ of water through a pipeline. The pressure difference between the inlet and outlet of the pump is equivalent to 10 m of water. What is the power transmitted to the water by the pump?

Solution.

Discharge, $Q = 2\ m^3\ s^{-1}$
Pressure difference between the inlet and outlet of the pump = 10 m of water
$\Rightarrow$ Increase in pressure head of water between exit of pump and entry to pump

$H = 10\ m$ of water

Power transmitted to water from the pump = γQH

$= 9810\ N\ m^{-3} \text{ x } 2\ m^3\ s^{-1} \text{ x } 10\ m$

$= 196200\ W = 196.2\ kW$

Topic: Energy Equation (Applications)