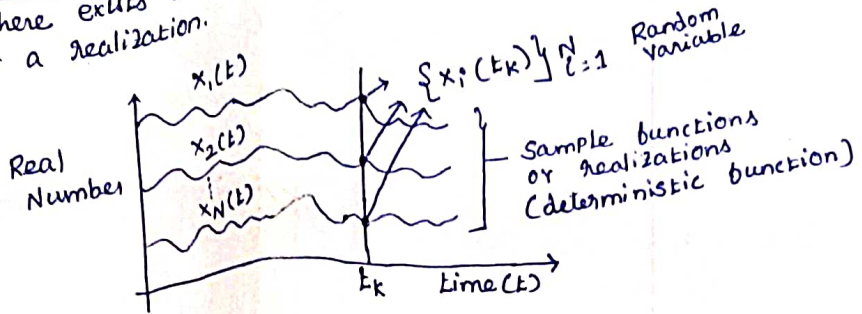


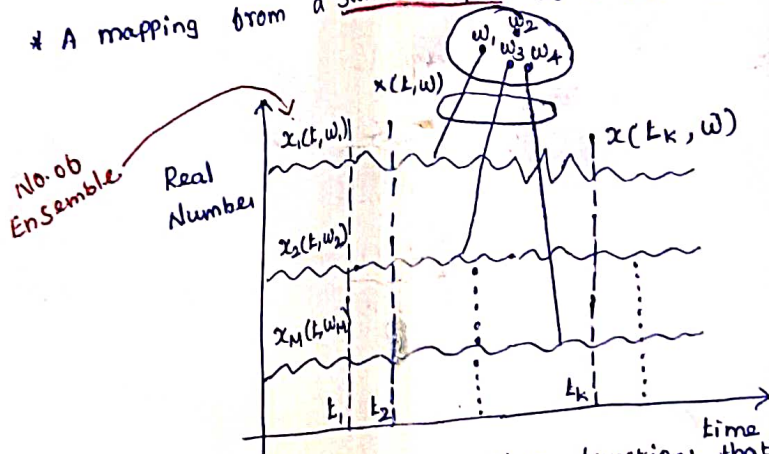
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RANDOM PROCESS:-

* A random process is a collection of time function, or signals, corresponding to various outcomes of a random experiment. For each outcome, there exists a deterministic function, which is called a sample function or a realization.



* A mapping from a sample space to a set of time functions.



* Ensemble:- The set of possible time functions that one sees.

* Denote this set by $X(t)$, where the time functions $x_1(t, w_1), x_2(t, w_2), x_3(t, w_3), \dots$ are specific members of the ensemble.

* At any time instant, $t = t_k$, we have random variable $x(t_k)$.

* At any two times instants, say t_1 and t_2 , we have two different random variable $x(t_1)$ and $x(t_2)$.

* Any relationship b/w any two random variable is called Joint PDF

Classification of Random Processes:-

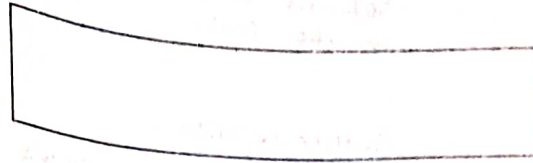
* Based on whether its statistics change with time:- the process is non-stationary (or) stationary.

* Different levels of stationary:-

✓ Strictly Stationary:- the joint pdf does not depend on any order & independent of a shift in time.

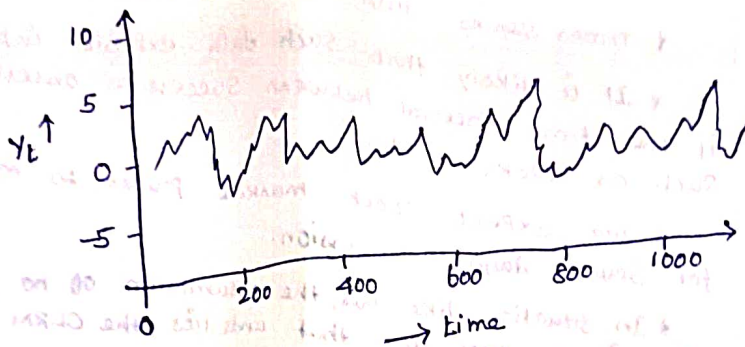
✓ Nth-order Stationary:- the joint pdf does not depend on the time shift, but depends on time spacing.

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STATIONARY PROCESS:-

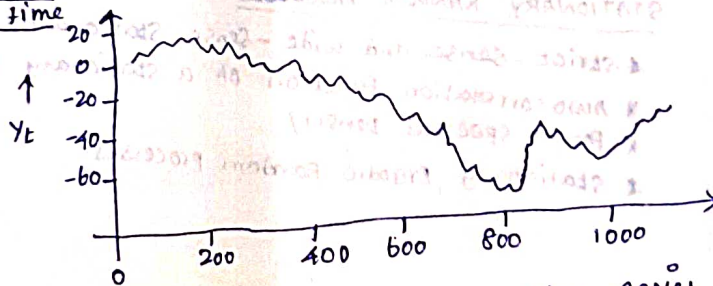
In mathematics and statistics, a stationary process (a.k.a. strict/strictly stationary process or strong/strongly stationary process) is a stochastic process whose (unconditional) joint probability distribution does not change when shifted in time.



Stationary Time Series

Non-Stationary Process

In contrast to the non-stationary process that has a variable variance and of mean that does not remain near or returns to a long-run mean over time



Non-Stationary Time Series

* An important type of non-stationary process that does not include a trend-like behavior is a cyclostationary process, which is a stochastic process that varies cyclically with time.

* Formally, let $\{x_t\}$ be a stochastic process and let $F_x(x_{t_1+\tau}, \dots, x_{t_k+\tau})$ represent the cumulative distribution function of the unconditional joint distribution of $\{x_t\}$ at times $t_1+\tau, \dots, t_k+\tau$. Then $\{x_t\}$ is said to be strictly stationary if, for all k , for all τ , and for all t_1, \dots, t_k

$$F_x(x_{t_1+\tau}, \dots, x_{t_k+\tau}) = F_x(x_{t_1}, \dots, x_{t_k})$$

→ Since τ does not affect $F_x(\cdot)$, F_x is not a function of time.

MEAN

Definition:-

Mean as a basic statistical measure is defined as an average value attained. with less time and resources available for calculation of complex or complicated measures, mean is considered desirable to get a quick, first hand estimate of the returns from the past.

Autocorrelation:-

* Autocorrelation occurs in time-series studies when the errors associated with a given time period carry over into future time period.

* For example, if we are predicting the growth of stock dividends, an overestimate in one year is likely to lead to overestimates in succeeding years.

* Time series data follow a natural ordering over time.

* It is likely that such data exhibit intercorrelation, especially if the time interval between successive observations is short, such as weeks or days.

* We expect stock market prices to move or move down for several days in succession.

* In situation like this, the assumption of no auto (or) serial correlation in the error term that underlies the CLRM will be violated.

* We experience autocorrelation when $E(u_i u_j) \neq 0$

STATIONARY RANDOM PROCESSES:-

* Strict-sense and wide-sense stationarity.

* Autocorrelation function of a stationary process

* Power spectral density

* Stationary Ergodic Random Processes.

Lecture Notes 7

Stationary Random Processes

- Strict-Sense and Wide-Sense Stationarity
- Autocorrelation Function of a Stationary Process
- Power Spectral Density
- Stationary Ergodic *Random Processes*



Stationary Random Processes

- Stationarity refers to time invariance of some, or all, of the statistics of a random process, such as mean, autocorrelation, n -th-order distribution
- We define two types of stationarity: strict sense (SSS) and wide sense (WSS)
- A random process $X(t)$ (or X_n) is said to be SSS if all its finite order distributions are time invariant, i.e., the joint cdfs (pdfs, pmfs) of $X(t_1), X(t_2), \dots, X(t_k)$ and $X(t_1 + \tau), X(t_2 + \tau), \dots, X(t_k + \tau)$ are the same for all k , all t_1, t_2, \dots, t_k , and all time shifts τ
- So for a SSS process, the first-order distribution is independent of t , and the second-order distribution — the distribution of any two samples $X(t_1)$ and $X(t_2)$ — depends only on $\tau = t_2 - t_1$

To see this, note that from the definition of stationarity, for any t , the joint distribution of $X(t_1)$ and $X(t_2)$ is the same as the joint distribution of $X(t_1 + (t - t_1)) = X(t)$ and $X(t_2 + (t - t_1)) = X(t + (t_2 - t_1))$



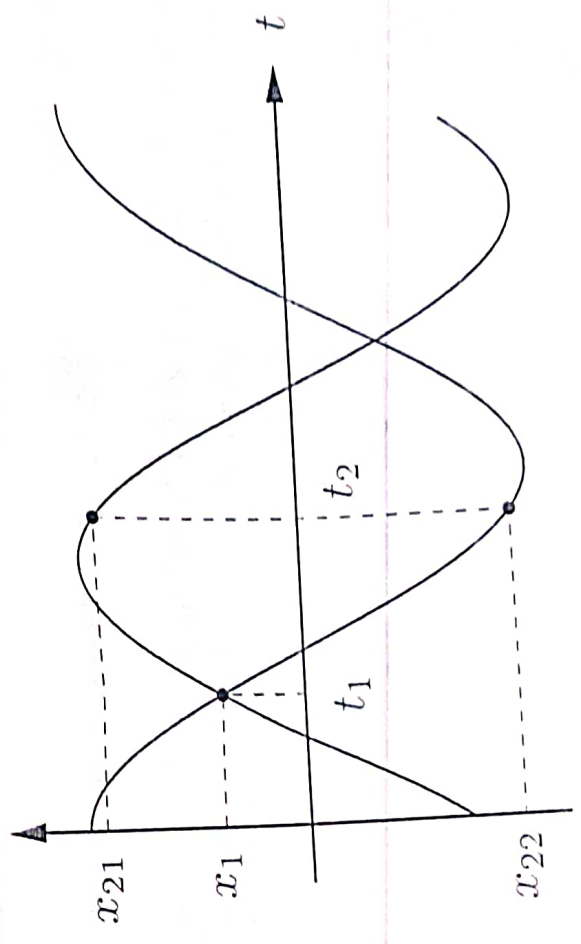
- Example: The random phase signal $X(t) = \alpha \cos(\omega t + \Theta)$ where $\Theta \in U[0, 2\pi]$ is SSS

◦ We already know that the first order pdf is

$$f_{X(t)}(x) = \frac{1}{\pi\alpha\sqrt{1 - (x/\alpha)^2}}, \quad -\alpha < x < +\alpha$$

which is independent of t , and is therefore stationary

- To find the second order pdf, note that if we are given the value of $X(t)$ at one point, say t_1 , there are (at most) two possible sample functions:



The second order pdf can thus be written as

$$\begin{aligned} f_{X(t_1), X(t_2)}(x_1, x_2) &= f_{X(t_1)}(x_1) \cdot f_{X(t_2)|X(t_1)}(x_2|x_1) \\ &= f_{X(t_1)}(x_1) \left(\frac{1}{2} \delta(x_2 - x_{21}) + \frac{1}{2} \delta(x_2 - x_{22}) \right), \end{aligned}$$

which depends only on $t_2 - t_1$, and thus the second order pdf is stationary

- Now if we know that $X(t_1) = x_1$ and $X(t_2) = x_2$, the sample path is totally determined (except when $x_1 = x_2 = 0$, where two paths may be possible), and thus all n -th order pdfs are stationary
- IID processes are SSS
- Random walk and Poisson processes are not SSS
- The Gauss-Markov process (as we defined it) is not SSS. However, if we set X_1 to the steady state distribution of X_n , it becomes SSS (see homework exercise)



Wide-Sense Stationary Random Processes

- A random process $X(t)$ is said to be wide-sense stationary (WSS) if its mean and autocorrelation functions are time invariant, i.e.,

- $E(X(t)) = \mu$, independent of t

- $R_X(t_1, t_2)$ is a function only of the time difference $t_2 - t_1$

- $E[X(t)^2] < \infty$ (technical condition)

- Since $R_X(t_1, t_2) = R_X(t_2, t_1)$, for any wide sense stationary process $X(t)$, $R_X(t_1, t_2)$ is a function only of $|t_2 - t_1|$

- Clearly $SSS \Rightarrow$ WSS. The converse is not necessarily true

A random process is called weak sense stationary (or) wide-sense stationary (WSS) if its mean function and its correlation func. do not change by shifts in time.



- Example: Let

$$X(t) = \begin{cases} +\sin t & \text{with probability } \frac{1}{4} \\ -\sin t & \text{with probability } \frac{1}{4} \\ +\cos t & \text{with probability } \frac{1}{4} \\ -\cos t & \text{with probability } \frac{1}{4} \end{cases}$$

◦ $E(X(t)) = 0$ and $R_X(t_1, t_2) = \frac{1}{2} \cos(t_2 - t_1)$, thus $X(t)$ is WSS

◦ But $X(0)$ and $X(\frac{\pi}{4})$ do not have the same pmf (different ranges), so the first order pmf is not stationary, and the process is not SSS

- For Gaussian random processes, WSS \Rightarrow SSS, since the process is completely specified by its mean and autocorrelation functions
- Random walk is not WSS, since $R_X(n_1, n_2) = \min\{n_1, n_2\}$ is not time invariant; similarly Poisson process is not WSS

Autocorrelation Function of WSS Processes

• Let $X(t)$ be a WSS process. Relabel $R_X(t_1, t_2)$ as $R_X(\tau)$ where $\tau = t_1 - t_2$

1. $R_X(\tau)$ is real and even, i.e., $R_X(\tau) = R_X(-\tau)$ for every τ

2. $|R_X(\tau)| \leq R_X(0) = E[X^2(t)]$, the "average power" of $X(t)$

This can be shown as follows. For every t ,

$$\begin{aligned}(R_X(\tau))^2 &= [E(X(t)X(t+\tau))]^2 \\ &\leq E[X^2(t)]E[X^2(t+\tau)] \quad \text{by Schwarz inequality} \\ &= (R_X(0))^2 \quad \text{by stationarity}\end{aligned}$$

3. If $R_X(T) = R_X(0)$ for some $T \neq 0$, then $R_X(\tau)$ is periodic with period T and so is $X(t)$ (with probability 1) !! That is,

$$R_X(\tau) = R_X(\tau + T), \quad X(\tau) = X(\tau + T) \text{ w.p.1 for every } \tau$$



- Example: The autocorrelation function for the periodic signal with random phase $X(t) = \alpha \cos(\omega t + \Theta)$ is $R_X(\tau) = \frac{\alpha^2}{2} \cos \omega \tau$ (also periodic)

- To prove property 3, we again use the Schwarz inequality: For every τ ,

$$\begin{aligned}
 [R_X(\tau) - R_X(\tau + T)]^2 &= [\mathbb{E}(X(t)(X(t + \tau) - X(t + \tau + T)))]^2 \\
 &\leq \mathbb{E}[X^2(t)] \mathbb{E}[(X(t + \tau) - X(t + \tau + T))^2] \\
 &= R_X(0)(2R_X(0) - 2R_X(T)) \\
 &= R_X(0)(2R_X(0) - 2R_X(0)) = 0
 \end{aligned}$$

Thus $R_X(\tau) = R_X(\tau + T)$ for all τ , i.e., $R_X(\tau)$ is periodic with period T

- The above properties of $R_X(\tau)$ are necessary but not sufficient for a function to qualify as an autocorrelation function for a WSS process



- The necessary and sufficient conditions for a function to be an autocorrelation function for a WSS process is that it be real, even, and nonnegative definite

By nonnegative definite we mean that for any n , any t_1, t_2, \dots, t_n and any real vector $\mathbf{a} = (a_1, \dots, a_n)$,

$$\sum_{i=1}^n \sum_{j=1}^n a_i a_j R(t_i - t_j) \geq 0$$

To see why this is necessary, recall that the correlation matrix for a random vector must be nonnegative definite, so if we take a set of n samples from the WSS random process, their correlation matrix must be nonnegative definite

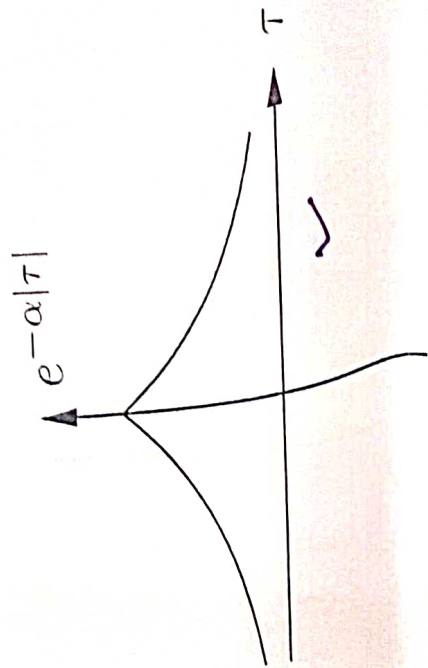
The condition is sufficient since such an $R(\tau)$ can specify a zero mean stationary Gaussian random process

- The nonnegative definite condition may be difficult to verify directly. It turns out, however, to be equivalent to the condition that the Fourier transform of $R_X(\tau)$, which is called the power spectral density $S_X(f)$, is nonnegative for all frequencies f

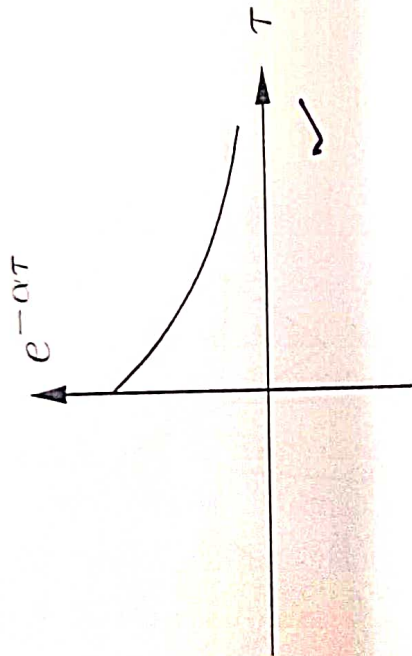


Which Functions Can Be an $R_X(\tau)$?

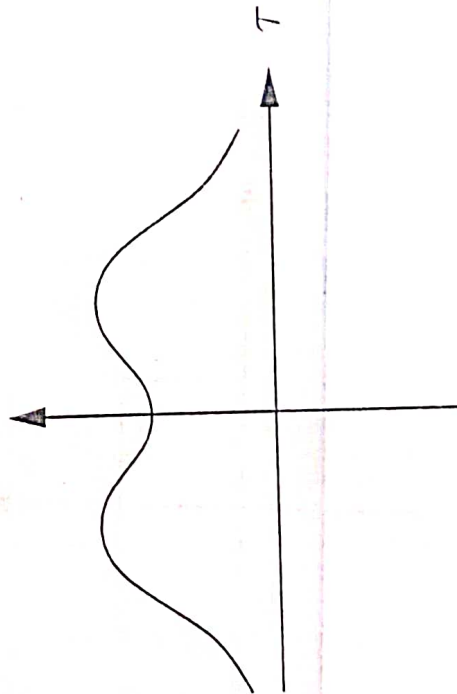
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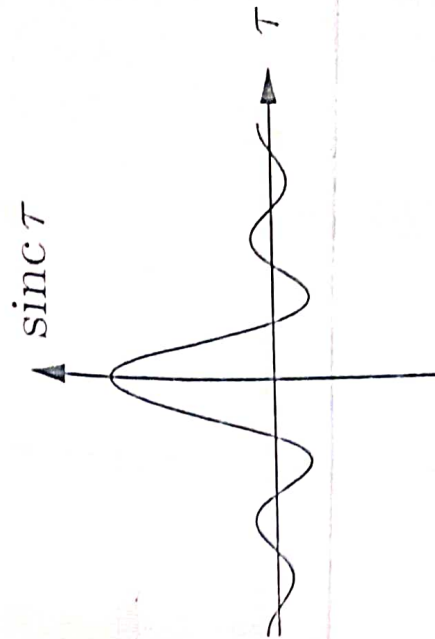
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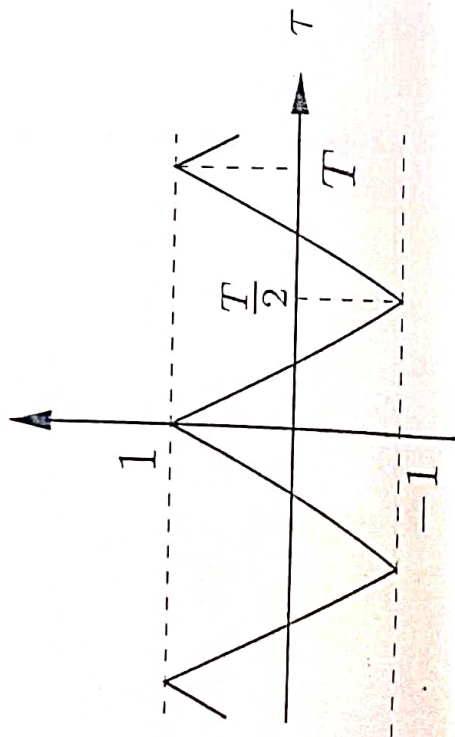


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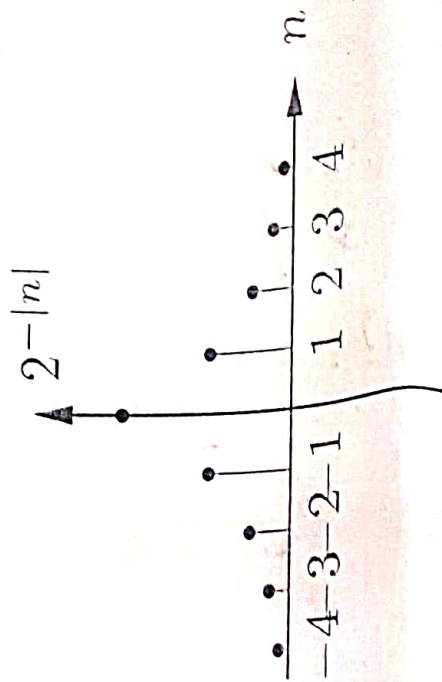


Which Functions can be an $R_X(\tau)$?

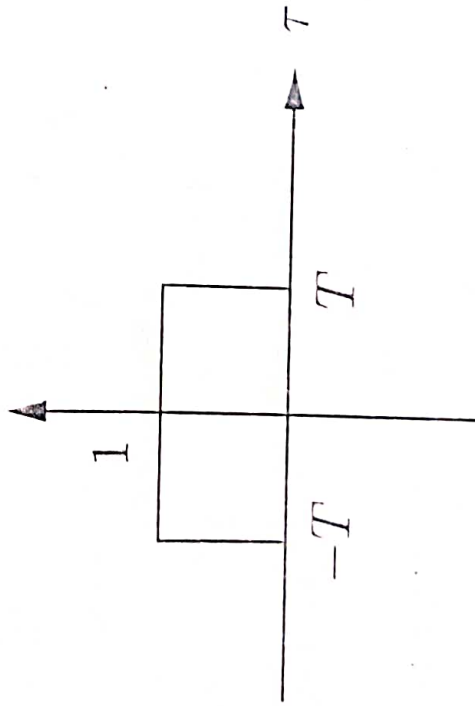
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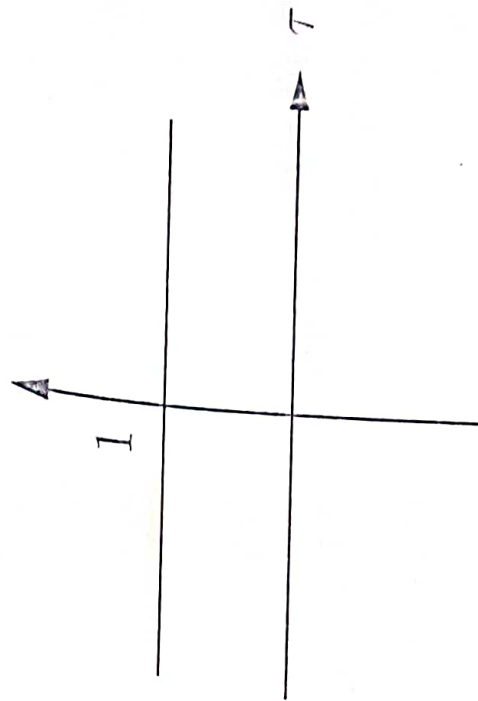
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Power Spectral Density

- The power spectral density (psd) of a WSS random process $X(t)$ is the Fourier transform of $R_X(\tau)$:

$$S_X(f) = \mathcal{F}(R_X(\tau)) = \int_{-\infty}^{\infty} R_X(\tau) e^{-i2\pi\tau f} d\tau$$

- For a discrete time process X_n , the power spectral density is the discrete-time Fourier transform (DTFT) of the sequence $R_X(n)$:

$$S_X(f) = \sum_{n=-\infty}^{\infty} R_X(n) e^{-i2\pi n f}, \quad |f| < \frac{1}{2}$$

- $R_X(\tau)$ (or $R_X(n)$) can be recovered from $S_X(f)$ by taking the inverse Fourier transform or inverse DTFT:

$$R_X(\tau) = \int_{-\infty}^{\infty} S_X(f) e^{i2\pi\tau f} df$$

$$R_X(n) = \int_{-\frac{1}{2}}^{\frac{1}{2}} S_X(f) e^{i2\pi n f} df$$

Properties of the Power Spectral Density

1. $S_X(f)$ is real and even, since the Fourier transform of the real and even function $R_X(\tau)$ is real and even
2. $\int_{-\infty}^{\infty} S_X(f) df = R_X(0) = E(X^2(t))$, the average power of $X(t)$, i.e., the area under S_X is the average power
3. $S_X(f)$ is the average power density, i.e., the average power of $X(t)$ in the frequency band $[f_1, f_2]$ is

$$\int_{-f_2}^{-f_1} S_X(f) df + \int_{f_1}^{f_2} S_X(f) df = 2 \int_{f_1}^{f_2} S_X(f) df$$

(we will show this soon)

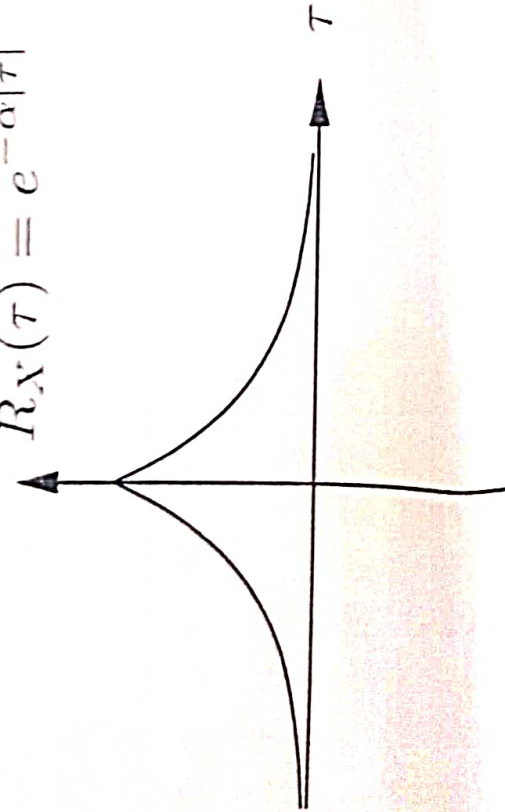
- From property 3, it follows that $S_X(f) \geq 0$. Why?
- In general, a function $S(f)$ is a psd if and only if it is real, even, nonnegative, and

$$\int_{-\infty}^{\infty} S(f) df < \infty$$

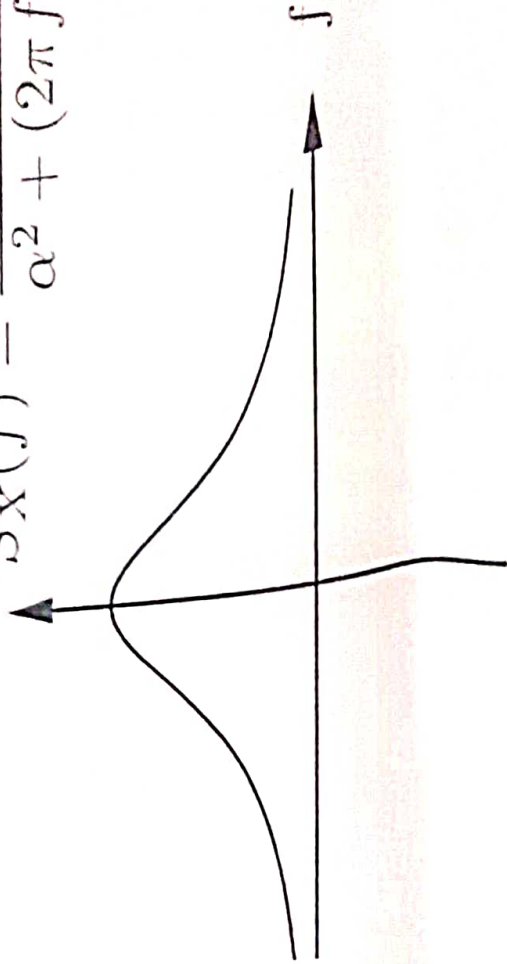
Examples

1.

$$R_X(\tau) = e^{-\alpha|\tau|}$$

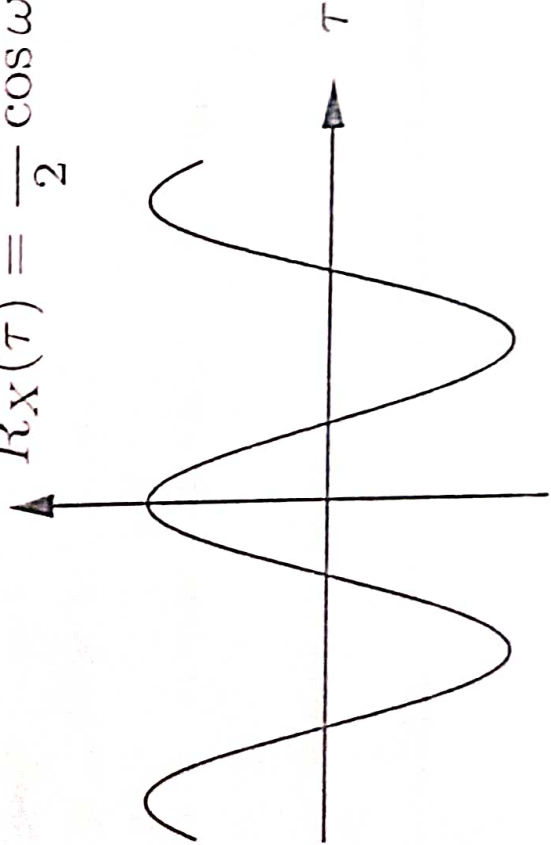


$$S_X(f) = \frac{2\alpha}{\alpha^2 + (2\pi f)^2}$$

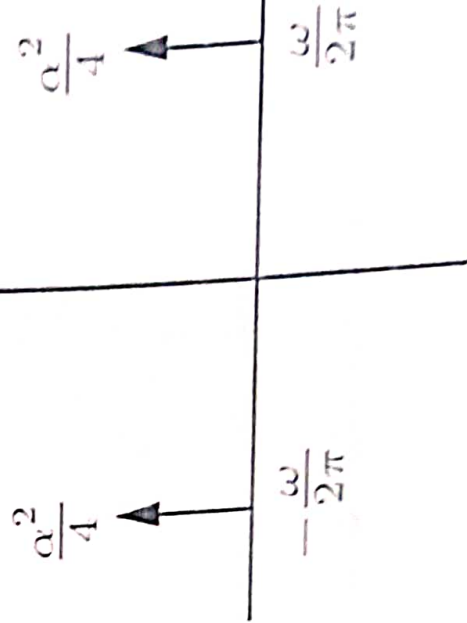


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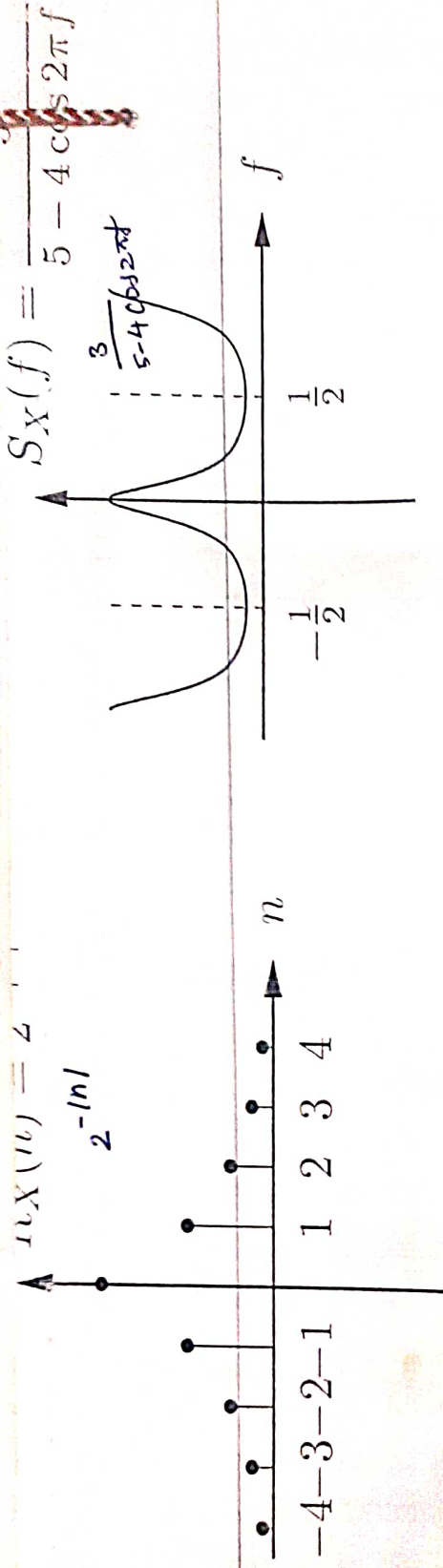
$$R_X(\tau) = \frac{\alpha^2}{2} \cos \omega \tau$$



$$S_X(f)$$

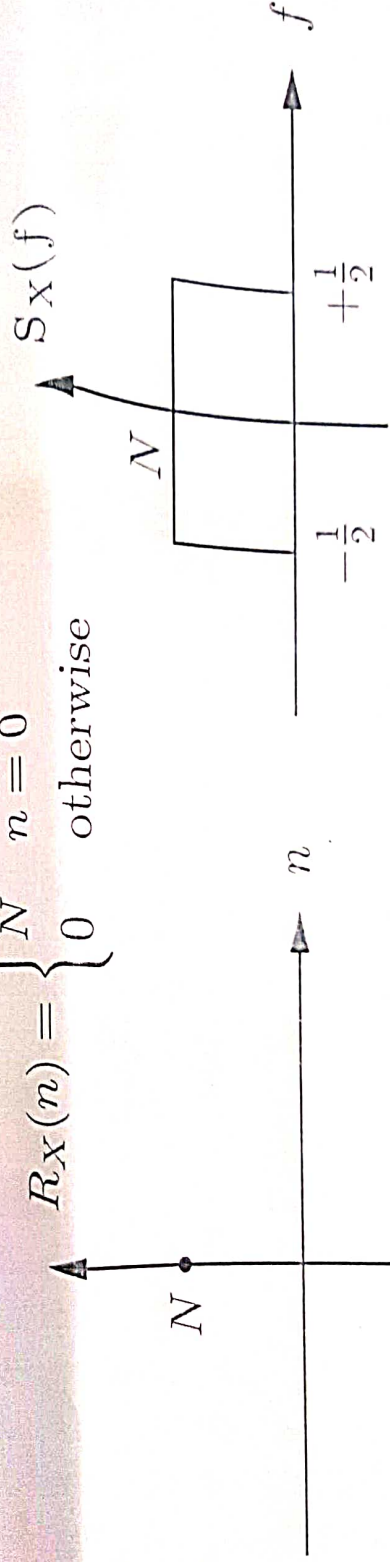


3.



4. Discrete time white noise process: $X_1, X_2, \dots, X_n, \dots$ zero mean, uncorrelated, with average power N

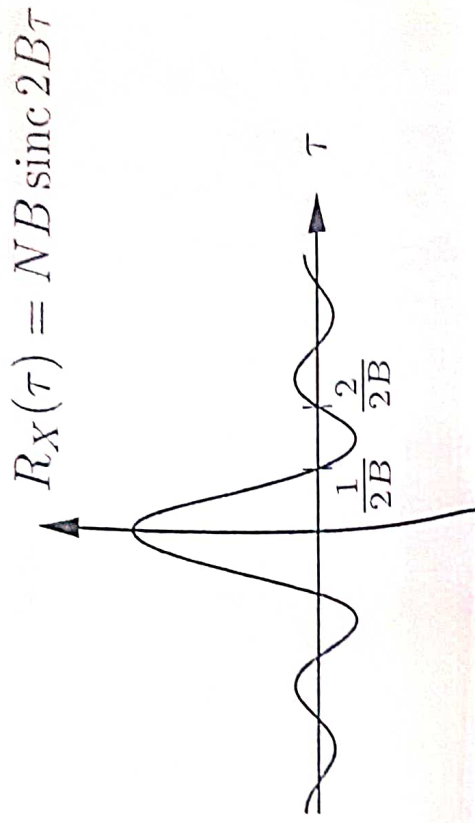
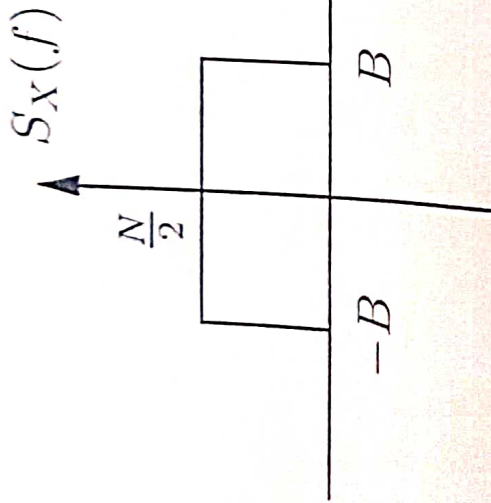
$$R_X(n) = \begin{cases} N & n = 0 \\ 0 & \text{otherwise} \end{cases}$$



If X_n is also a GRP, then we obtain a discrete time WGN process



5. Bandlimited white noise process: WSS zero mean process $X(t)$ with



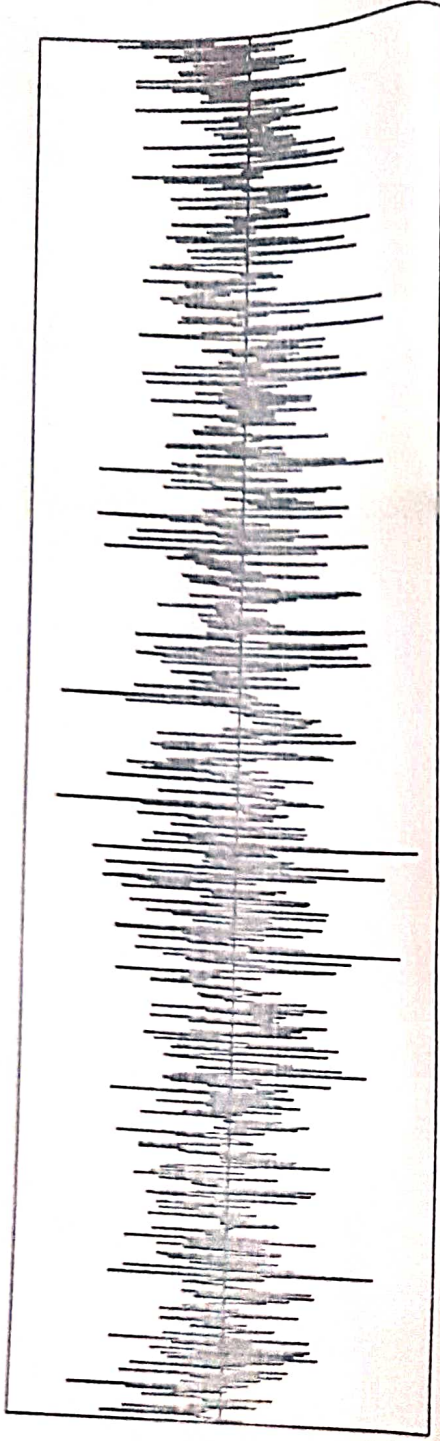
For any t , the samples $X\left(t \pm \frac{n}{2B}\right)$ for $n = 0, 1, 2, \dots$ are uncorrelated

6. White noise process: If we let $B \rightarrow \infty$ in the previous example, we obtain a white noise process, which has

$$S_X(f) = \frac{N}{2} \quad \text{for all } f$$

$$R_X(\tau) = \frac{N}{2} \delta(\tau)$$

If, in addition, $X(t)$ is a GRP, then we obtain the famous white Gaussian noise (WGN) process



- Remarks on white noise:
 - For a white noise process, all samples are uncorrelated
 - The process is not physically realizable, since it has infinite power
 - However, it plays a similar role in random processes to point mass in physics and delta function in linear systems
 - Thermal noise and shot noise are well modeled as white Gaussian noise, since they have very flat psd over very wide band (GHz)

No Need

X

Stationary Ergodic Random processes

- Ergodicity refers to certain time averages of random processes converging to their respective statistical averages
- We focus only on mean ergodicity of WSS processes
- Let $X_n, n = 1, 2, \dots$, be a discrete time WSS process with mean μ and autocorrelation function $R_X(n)$
- To estimate the mean of X_n , we form the sample mean

$$\bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i$$

- The process X_n is said to be mean ergodic if:
$$\bar{X}_n \rightarrow \mu \quad \text{in mean square,}$$

i.e., $\lim_{n \rightarrow \infty} E[(\bar{X}_n - \mu)^2] = 0$
- Since $E(\bar{X}_n) = \mu$, this condition is equivalent to:

$$\text{Var}(\bar{X}_n) \rightarrow 0 \text{ as } n \rightarrow \infty$$

- We can express this condition in terms of $C_X(n) = R_X(n) - \mu^2$ as follows

$$\begin{aligned} \text{Var}(\bar{X}_n) &= \frac{1}{n^2} \sum_{i=1}^n \sum_{j=1}^n \mathbb{E}[(X_i - \mu)(X_j - \mu)] \\ &= \frac{1}{n^2} \sum_{i=1}^n \sum_{j=1}^n C_X(i-j) \\ &= \frac{1}{n} C_X(0) + \frac{2}{n^2} \sum_{i=1}^{n-1} (n-i) C_X(i) \end{aligned}$$

Since by definition, $C_X(0) < \infty$, the condition for mean ergodicity is:

$$\frac{2}{n^2} \sum_{i=1}^{n-1} (n-i) C_X(i) \rightarrow 0$$

- Example: Let X_n a WSS process with $C_X(n) = 0$ for $n \neq 0$, then the process is mean ergodic
- The process does not need to have uncorrelated samples for it to be mean ergodic, however (see stationary Gauss-Markov process problem in HW7)

- Not every WSS process is mean ergodic

Example: Consider the coin with random bias P example in Lecture Notes 5.

The random process X_1, X_2, \dots is stationary

However, it is not mean ergodic, since $\bar{X}_n \rightarrow P$ in m.s.

- Remarks:
 - The process in the above example can be viewed as a mixture of IID Bernoulli(p) processes, each of which is stationary ergodic (it turns out that every SSS process is a mixture of stationary ergodic processes)
 - Ergodicity can be defined for general (not necessarily stationary) processes (this is beyond the scope of this course)

Mean Ergodicity for Continuous-time WSS Processes

- Let $X(t)$ be WSS process with mean μ and autocorrelation function $R_X(\tau)$
- To estimate the mean, we form the time average

$$\bar{X}(t) = (1/t) \int_0^t X(\tau) d\tau$$

But what does this integral mean?

- Integration of RP: Let $X(t)$ be a RP and $h(t)$ be a function. Define the integral

$$\int_a^b h(t)X(t)dt$$

as the limit of a sum (as in Riemann integral of a deterministic function) in m.s.
Let $\Delta > 0$ such that $b - a = n\Delta$ and $a \leq \tau_1 \leq a + \Delta \leq \tau_2 \leq a + 2\Delta \leq \dots \leq \tau_{n-1} \leq a + (n-1)\Delta \leq \tau_n \leq a + n\Delta = b$, then the above integral exists if the Riemann sum

$$\sum_{i=1}^{n-1} h(\tau_i)X(\tau_i)\Delta$$

has a limit in m.s.



Moreover, if the random integral exists for all a, b , then we can define

$$\int_{-\infty}^{\infty} h(t)X(t)dt = \lim_{a, b \rightarrow \infty} \int_a^b h(t)X(t)dt \quad \text{in m.s.}$$

Key fact: The existence of the m.s. integral depends only on R_X and h

More specifically, the above integral exists iff

$$\int_a^b \int_a^b R_X(t_1, t_2)h(t_1)h(t_2)dt_1dt_2$$

exists (in the normal sense)

- Now let's go back to mean ergodicity for continuous-time WSS process
By definition, mean ergodicity means that

$$\lim_{t \rightarrow \infty} E[(\bar{X}(t) - \mu_X)^2] \rightarrow 0$$

Since $E(\bar{X}(t)) = \mu_X$, the condition for mean ergodicity is the same as

$$\lim_{t \rightarrow \infty} \text{Var}(\bar{X}(t)) = 0$$

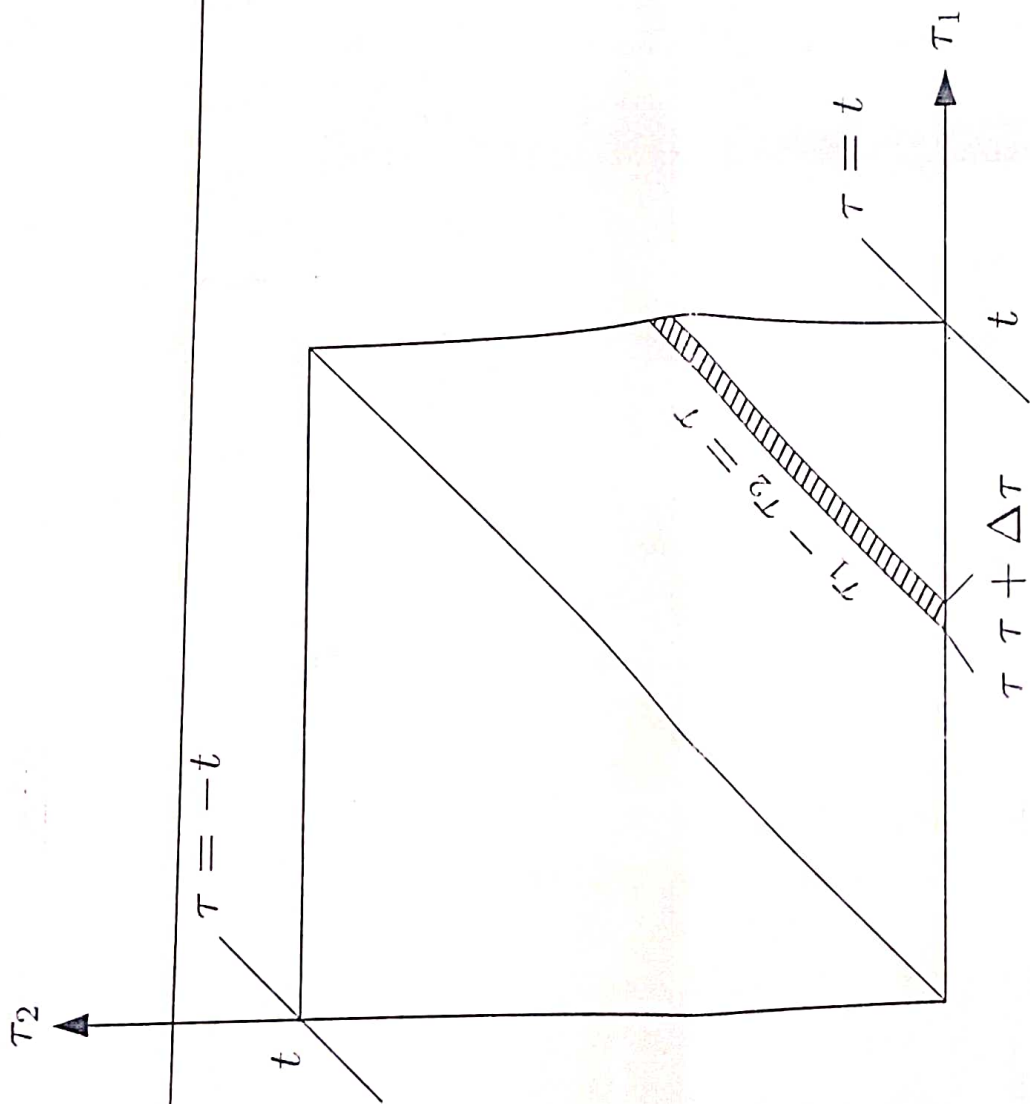
Now, consider

$$\begin{aligned} E(\bar{X}^2(t)) &= E \left[\left(\frac{1}{t} \int_0^t X(\tau) d\tau \right)^2 \right] \\ &= E \left(\frac{1}{t^2} \int_0^t \int_0^t X(\tau_1) X(\tau_2) d\tau_1 d\tau_2 \right) \\ &= \frac{1}{t^2} \int_0^t \int_0^t R_X(\tau_1, \tau_2) d\tau_1 d\tau_2 \\ &= \frac{1}{t^2} \int_0^t \int_0^t R_X(\tau_1 - \tau_2) d\tau_1 d\tau_2 \end{aligned}$$

From the figure below, the double integral reduces to the single integral

$$E(\bar{X}^2(t)) = \frac{2}{t^2} \int_0^t (t - \tau) R_X(\tau) d\tau$$





- Hence, a WSS process $X(t)$ is mean ergodic iff

$$\lim_{t \rightarrow \infty} \frac{2}{t^2} \int_0^t (t - \tau) R_X(\tau) d\tau = \mu_X^2$$

Gaussian process:-

AU
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* In Probability theory and Statistics, a Gaussian process is a stochastic process (a collection of random variables indexed by time or state), such that every finite collection of those random variables has a multivariate normal distribution.

* The distribution of a Gaussian process is the joint distribution of all those random variables, and as such, it is a distribution over functions with a continuous domain. Example:- Time (or) state

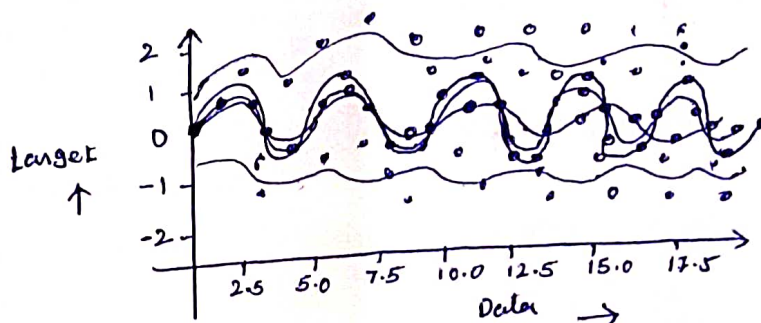
* Gaussian processes are useful in statistical modelling, benefiting from properties inherited from the normal.
- For example, if a random process is modelled as a Gaussian process, the distributions of various derived quantities can be obtained explicitly.

* A time continuous stochastic process is Gaussian if and only if for every finite set of indices t_1, \dots, t_k in the index set 'T'

$$X_{t_1}, \dots, X_{t_k} = (X_{t_1}, \dots, X_{t_k})$$

is a multivariate Gaussian random variable

(i.e.) Every linear combination of $(X_{t_1}, \dots, X_{t_k})$ has a univariate normal distribution. using characteristic functions of random variable,



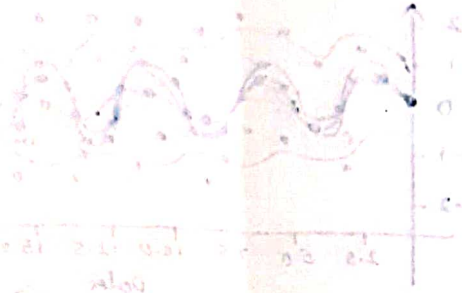
Gaussian Process Regression (Prediction) compared with other regression models

• Example: Let $X(t)$ be a WSS with zero mean and $R_X(\tau) = e^{-|\tau|}$.

Evaluating the condition on mean ergodicity, we obtain

$$\frac{2}{t^2} \int_0^t (t-\tau) R_X(\tau) d\tau = \frac{2}{t^2} (e^{-t} + t - 1),$$

which $\rightarrow 0$ as $t \rightarrow \infty$. Hence $X(t)$ is mean ergodic



Noise:-

Introduction:-

* Undesired electrical signals which are introduced with a message signal during the transmission, or processing of the latter, are called noise

* Noise, thus, is an unwanted signal that corrupts a desired message signal.

* In general, noise may be predictable, or unpredictable in nature.

* The predictable noise can be estimated, and eliminated by proper engineering design.

Example:- Power supply hum, spurious oscillation in feedback amplifiers

* Predictable noise, generally is man-made, and can be reduced or eliminated.

* unpredictable noise varies randomly with time and, as such we have no control over this noise.

* Identification of the message signal at the receiver depends upon the amount of noise accompanied by the message during the process of communication.

* In the absence of noise, identification of the message signal at the receiver is perfect.

* The presence of noise complicates the system of communication.

Sources of Noise:-

There are various sources of random noise. They are broadly classified as,

- ✓ (a) External Noise.
- ✓ (b) Internal Noise.

The External noise is created outside the circuit, and includes:-

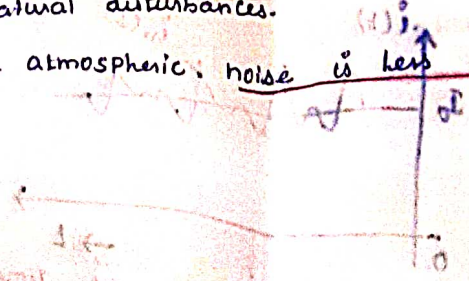
(i) Erratic Natural Disturbances:-

* This type of noise does not occur regularly. It is caused by lightning, electrical storms and other atmospheric disturbances.

* This noise is unpredictable in nature, and is known as atmospheric (or) static noise.

* Besides this, the extraterrestrial noise is also created by erratic natural disturbances.

* The atmospheric noise is less severe above 30MHz



(ii) Man-made Noise:-

* This noise is because of the undesired pick-ups from electrical appliances, such as motors, switch gears, auto-mobile and aircraft ignitions, etc...

* This type of noise is under human control and can be eliminated by removing the source of the noise.

* This noise is effective in frequency range of 1 MHz - 500 MHz

* It is caused by spontaneous fluctuations in the physical system.

* The fluctuation noise is very significant, and will be treated in greater detail.

* The two important types of fluctuation noise are

✓ (i) shot noise

✓ (ii) thermal noise.

Shot Noise:-

* Shot noise is random fluctuation that accompanies any direct current crossing potential barrier.

* The effect occurs because the carriers simultaneously but rather with random distribution in the timing of each carrier, which gives rise to random component of current superimpose on the steady current.

* In case of bipolar junction transistors, the bias current crossing the forward biased emitter base junction carries the shot noise.

* When amplified, this noise sounds as though a shower of lead shots were balling on a metal sheet, hence the name shot noise.

* Although it is always present, shot noise is not normally observed during measurement of direct current because it is small compared to the DC value, however it does contribute significantly to the noise in amplifier circuit.

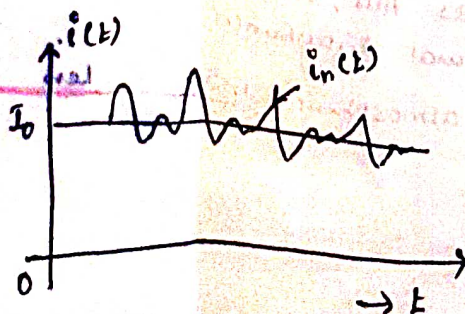
* The mean square noise component is proportion to the DC flowing, and for most devices the mean-square, shot noise is given by.

$$I_n^2 = 2 I_{dc} q_e B_n \text{ ampere}^2$$

where I_{dc} is the direct current in ampere's

q_e is the magnitude of electronic charge

B_n is the equivalent noise B.W in Hertz.



Current variation with time in shot noise.

"Due to the random behaviour of charge carriers (Electrons and holes)"



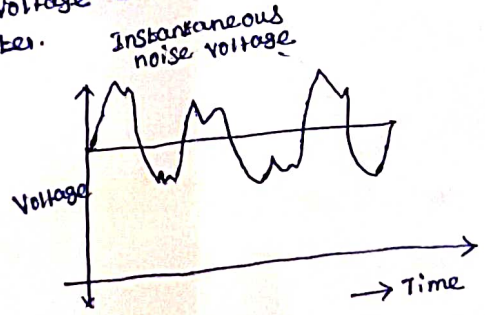
RESISTOR NOISE :- (or) JOHNSON NOISE (or) THERMAL NOISE

Thermal Noise generated in a resistor (or) a resistive component

- * The noise arising due to random motion of free charged particles (usually electrons) in a conducting media, such as a resistor, is called resistor noise. This noise is also known as Johnson noise
- * The random agitation is a universal phenomenon at atomic level and is caused by the energy supplied through flow of heat. The intensity of random motion is proportional to thermal (heat) energy supplied and is zero at a temperature of absolute zero. This noise is also known as thermal noise.

CAUSE :-

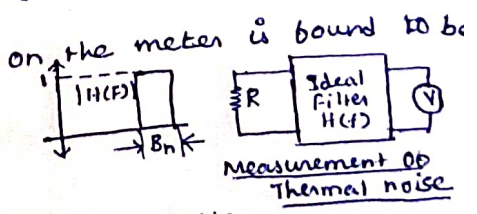
- The free electrons within an electrical conductor possess kinetic energy as a result of heat exchange b/w the conductor and its surroundings.
- Due to this kinetic energy the electrons are in motion, this motion is randomized through collisions with imperfections in the structure of the conductor. This process occurs in all real conductors and gives rise to conductor's resistance.
- As a result, the electron density throughout the conductor varies randomly, giving rise to randomly varying voltage across the ends of conductor. Such voltage can be observed as flickering on a very sensitive Voltmeter.



- The average or mean noise voltage across the conductor is zero, but the root-mean-square value is finite and can be measured.
- The mean square value of the noise voltage is proportional to the resistance of the conductor, to its absolute temperature, to the frequency bandwidth of the device measuring the noise.

- The mean-square voltage measured on the meter is found to be

$E_n^2 = 4RKT B_n \rightarrow \text{①}$



where

- E_n - root-mean-square noise voltage, volts
- R - resistance of the conductor, ohms
- T - conductor temperature, kelvins
- B_n - noise bandwidth, hertz
- K - Boltzmann constant (1.38×10^{-23} J/K)

and the rms noise voltage is given by

$E_n = \sqrt{4RKT B_n}$

WHITE NOISE:

* White light contains all color frequency. In the same way white noise, too contains all frequency.

* The Power density spectrum of a white noise is constant for all frequency, which means it contains all the frequency components in equal amount.

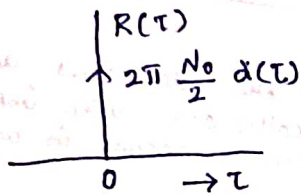
* When the probability of occurrence of a white noise level is specified by a Gaussian distribution function it is known as white Gaussian noise.

* The power density spectrum of that noise is independent of the operating frequency.

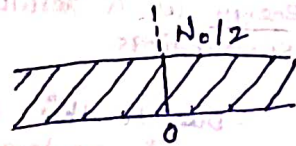
* Hence, shot noise and thermal noise may be considered as white Gaussian noise for all practical purpose.

* The power density of white noise is

$$S_w(\omega) = N_0/2$$



(a) Autocorrelation



(b) Power spectrum of white noise.

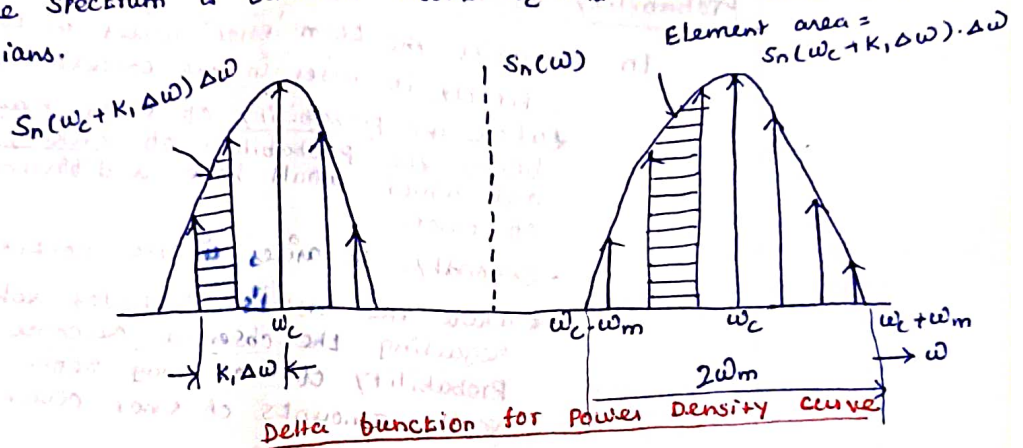


Narrow-band Noise:-

- * In communication system, message signals intermixed with noise are usually passed through bandpass filters.
- * The bandpass filters have narrow bandwidths in the sense that the bandwidth is small as compared to centre frequency.
- * The wideband noise accompanied with the desired signal is also passed through this bandpass filter.
- * Hence, in general, we have to deal with bandpass noise for evaluating the noise performance of a communication system
- * we will develop a mathematical model of this narrow-band noise

- Consider a narrow band Gaussian noise $n(t)$ with spectral density function $S_n(\omega)$

- The spectrum is centered about ω_c and has a bandwidth of $2\omega_m$ radians.



* Power density curve may be approximated by a set of delta function as shown in figure.

- Element area given by $S_n(\omega_c + k, \Delta\omega) \cdot \Delta\omega$ becomes a delta function under the limit $\Delta\omega \rightarrow 0$

- The pair of delta function corresponding to this element area will be located at $\omega = \pm(\omega_c - k, \Delta\omega)$

- A strength equal to the area $S_n(\omega_c + k, \Delta\omega) \cdot \Delta\omega$

* The corresponding pair of Delta function is denoted by

$$\{ S_n(\omega_c + k, \Delta\omega) \Delta\omega \} \delta[\omega \pm (\omega_c - k, \Delta\omega)]$$

* Similarly, a pair of Delta function located at $\pm(\omega_c + k, \Delta\omega)$ is given by

$$\{ S_n(\omega_c + k, \Delta\omega) \cdot \Delta\omega \} \delta[\omega \pm (\omega_c + k, \Delta\omega)]$$

* The characteristic curve may be approximated by summation of similar delta function located at a uniform spacing of $\Delta\omega$.

- The entire curve can be represented in terms of delta function as follows.

$$S_n(\omega) = \lim_{\Delta\omega \rightarrow 0} \sum_k \{ S_n(\omega_c + k\Delta\omega) \Delta\omega \} [\delta(\omega - \omega_c - k\Delta\omega) + \delta(\omega + \omega_c + k\Delta\omega)]$$

Signal to Noise Ratio:-

* The ratio of a signal power to the accompanying noise power is referred as signal to noise ratio and is denoted by S/N

* If a signal voltage $V_s(t)$ is accompanied by a noise voltage source $V_n(t)$, the ratio of signal to noise power is

$$\frac{S}{N} = \frac{V_s^2}{V_n^2}$$

* In terms of power density as follows:-

$$\frac{S}{N} = \frac{S_s(\omega)}{S_n(\omega)} = \frac{\text{Power density spectrum of signal voltage}}{\text{Power density spectrum of noise voltage}}$$

* The S/N ratio is always referred to the power ratio, unless stated otherwise. Sometimes, the ratio of a signal voltage (rms) to a noise voltage (rms) is also specified, but in that case, it is specifically stated that S/N voltage ratio is being considered.

Probability of Error

In statistics, the term 'error' arises in two ways.

- Firstly, it arises in the context of decision making, where the probability of error may be considered as being the probability of making a wrong decision and which would have a different value of each type of error.

- Secondly, it arises in the context of statistical modelling where the model's predicted value may be in error regarding the observed outcome and where the term probability of error may refer to the probabilities of various amounts of error occurring.

Noise Temperature

* Noise temperature is a means for specifying noise in terms of an equivalent temperature

* Noise power is directly proportional to temperature in degree Kelvin and that

* Note that the equivalent noise temperature (T_e) is not the physical temperature of the amplifier, but rather a theoretical construct that is an equivalent temperature that produces that amount of noise power.

* The noise temperature is related to the noise factor by

$$T_e = (F_n - 1) T_0$$

and to noise figure by

$$T_e = \left[\text{antilog} \left(\frac{NF}{10} \right) - 1 \right] K T_0$$

Now that we have noise temperature T_e , we can also define noise factor and noise figure in terms of noise temp.

$$F_n = \frac{T_e}{T_0} + 1$$

and

$$NF = 10 \log \left(\frac{T_e}{T_0} + 1 \right)$$

The total noise in any amplified (or) network is the sum of internally and externally generated noise. In terms of noise temperature

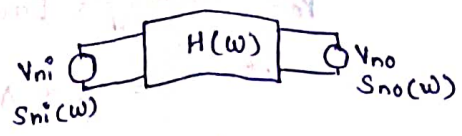
$$P_n(\text{total}) = G k B (T_0 + T_e)$$

$P_n \rightarrow$ Total noise power
 $G \rightarrow$ amplifier gain

$k \rightarrow$ Boltzmann's constant
 $B \rightarrow$ N/w Bandwidth in Hz
 $T_0 \rightarrow 290 \text{ K}$

Noise bandwidth:-

- The noise bandwidth is an important parameter for specifying the noise power at the output of a band pass linear system
- Linear band pass system



- The square of transfer func.
 - The noise power at the output of the system is given by
- $$P_o = V_{no}^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{ni}(w) |H(w)|^2 dw = \frac{1}{\pi} \int_0^{\infty} S_{ni}(w) |H(w)|^2 dw$$

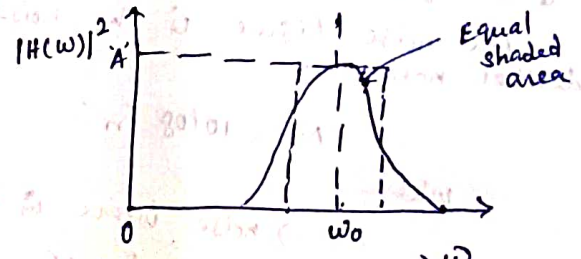
- For all practical purposes, the input noise power density is assumed to be constant with frequency. Let us take this constant value as 'C'. Thus

$$S_{ni}(w) = C$$

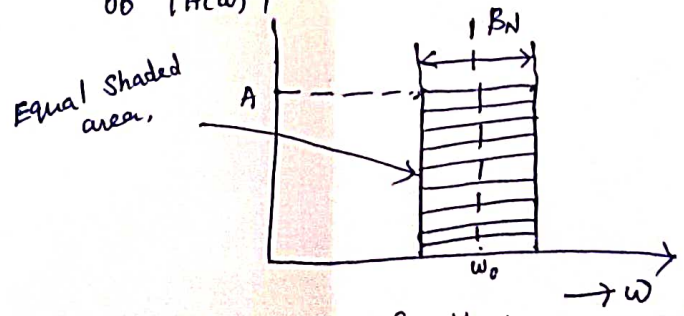
- Then the output noise power is given by

$$P_o = \frac{C}{\pi} \int_0^{\infty} |H(w)|^2 dw$$

- The integral in the right-hand side is the area under the curve square of transfer function (ie) plot of $|H(w)|^2$



- Ideal band pass system with rectangular characteristics of $|H(w)|^2$



- The area under this ideal rectangular characteristic specifying the power of the signal is given by

$$A \times B_n$$

where

$$A = |H(w_0)|^2$$

Equating the areas of actual and ideal systems,

$$A \times B_N = \int_0^\infty |H(\omega)|^2 d\omega$$

noise bandwidth B_N as

$$B_N = \frac{1}{A} \int_0^\infty |H(\omega)|^2 d\omega$$

The noise power, in terms of B_N is obtained as follows

$$P_o = V_{no}^2 = \frac{C}{\pi} \times [\text{Area under the curve } |H(\omega)|^2 \text{ of actual system}]$$
$$= \frac{C}{\pi} \times [A B_N]$$

where

The bandwidth of the ideal system is called as equivalent noise bandwidth denoted by B_N

$$P_o = \frac{C A B_N}{\pi}$$

$C \rightarrow$ constant equal to the power density spectrum.

Note

Equivalent noise B.W is the bandwidth of that ideal bandpass system which produces the same noise power as the actual system.

Noise figure:-

The noise figure is a frequently used measure of an amplifier's goodness, or its departure from the ideal. Thus it is a figure of merit.

The noise figure is the noise factor converted to decibel notation.

$$NF = 10 \log F_n$$

where

$NF \rightarrow$ noise figure in decibels (dB)

$F_n \rightarrow$ noise factor.



Amplitude Modulation system

Introduction:-

Amplitude Modulation is a process by which the amplitude of the carrier signal is varied in accordance with the instantaneous value of the modulating signal, but frequency and phase of carrier wave is remains constant.

Amplitude Modulation can be classified as follows:

- (i) Amplitude Modulation
- (ii) Frequency Modulation
- (iii) Phase Modulation

Amplitude Modulation and Phase Modulation are collectively called as Angle Modulation.

Let a sinusoidal carrier wave in analog modulation is given by,

$$V_c(t) = V_c \sin(\omega_c t + \theta) \rightarrow \textcircled{1}$$

$$\text{or, in general } V_c(t) = A \sin(\omega_c t + \theta) \rightarrow \textcircled{2}$$

Where, $A = V_c = \text{Amplitude of the carrier signal}$
 $\omega_c = \text{Angular Frequency.}$

$\theta = \text{Phase angle.}$

The signals usually contain three parameters, amplitude, frequency and Phase angle in Modulation, any one of these parameters may be

① $V_c(t) = V_c \sin(\omega_c t + \theta)$



varied in accordance with the baseband (or) message signal accordingly, the modulation process is termed as

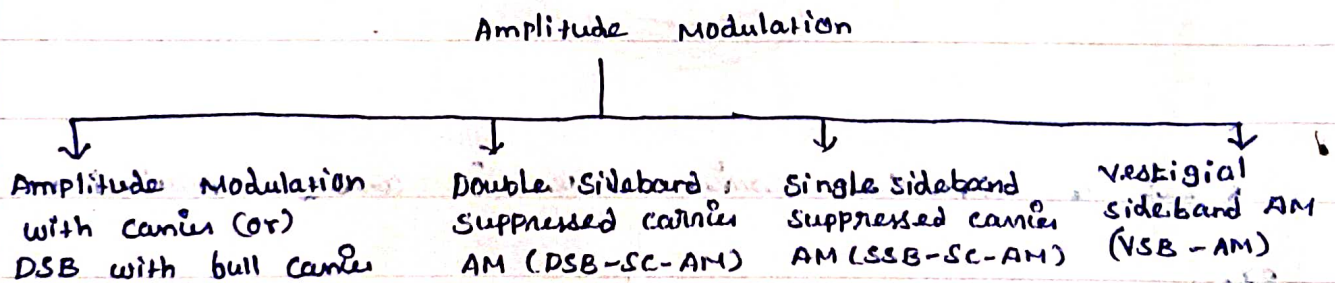
- (i) Amplitude Modulation
- (ii) Frequency Modulation
- (iii) Phase Modulation

* The frequency and phase modulation are combinedly called as Angle Modulation.

* Amplitude Modulation

- Amplitude Modulation is the process by which amplitude of the carrier signal is varied in accordance with the instantaneous value of the modulating signal, but frequency and phase of carrier wave is remains constant.

- Amplitude Modulation can be classified as follows:-



(i) AMPLITUDE MODULATION \rightarrow changing the amplitude of the carrier signal

* Let the modulating signal and carrier signal can be written as

$$V_m(t) = V_m \sin \omega_m t \rightarrow (3)$$

$$V_c(t) = V_c \sin \omega_c t \rightarrow (4)$$

According to the definition, the amplitude of the carrier signal is changed after modulation,

$$V_{AM} = V_c + V_m(t) = V_c + V_m \sin \omega_m t \rightarrow (5)$$

$$= V_c \left[1 + \frac{V_m}{V_c} \sin \omega_m t \right] = V_c (1 + m_a \sin \omega_m t) \rightarrow (6)$$

where

$$m_a = V_m / V_c \rightarrow \text{Modulation Index}$$



ANALOG COMMUNICATION

* The instantaneous amplitude of modulated signal (or) AM envelope can be written as

$$V_{AM}(t) = V_{AM} \sin \omega_c t \rightarrow (7)$$

Subst. the value of V_{AM} in equation (7)

$$V_{AM}(t) = V_c (1 + m_a \sin \omega_m t) \cdot \sin \omega_c t$$

$$= V_c \sin \omega_c t + V_c m_a \sin \omega_m t \cdot \sin \omega_c t \rightarrow (8)$$

We know

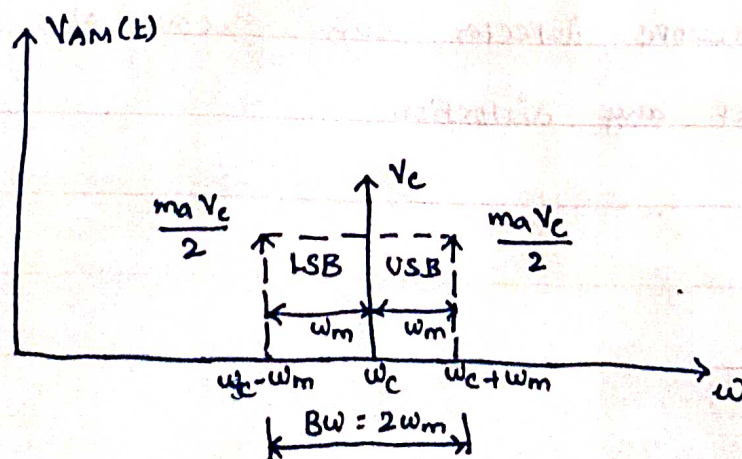
$$\sin \omega_m t \cdot \sin \omega_c t = \frac{\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t}{2}$$

$$V_{AM}(t) = V_c \sin \omega_c t + \frac{V_c m_a}{2} [\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t] \rightarrow (9)$$

Eq (9) of the amplitude modulated wave contains three frequency components.

1. carrier frequency ω_c of amplitude V_c
2. upper side band ($\omega_c + \omega_m$) having amplitude $\frac{m_a V_c}{2}$
3. Lower side band ($\omega_c - \omega_m$) having amplitude $\frac{m_a V_c}{2}$

* The frequency component graphical representation as shown in figure.





DEGREES OF MODULATION:-

- The modulating signal is preserved from the envelope of amplitude modulation modulated signal only if $V_m < V_c$ then $m_a < 1$

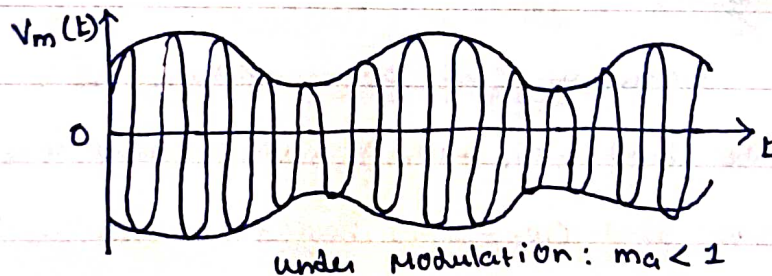
where V_m = maximum amplitude of modulating signal

V_c = maximum amplitude of carrier signal

- There are three degree of modulation depending upon the amplitude of the message signal relative to carrier amplitude.

1) under modulation $V_m < V_c$ then $m_a < 1$

* Hence the envelope of amplitude modulated signal does not reach the zero amplitude axis and shown in figure.



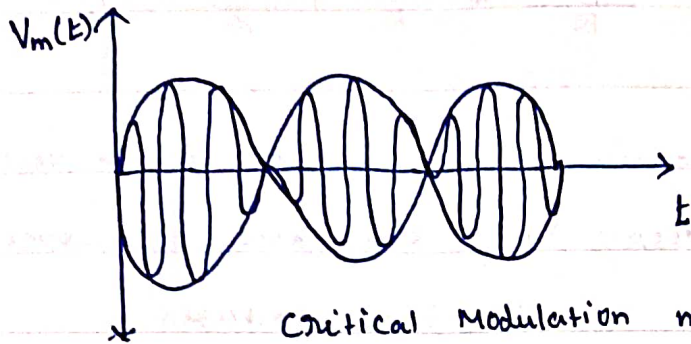
* Message signal is fully preserved from the envelope of the AM wave. This is known as under modulation.

* Envelope detector can recover the message signal without any distortion.



2. Critical Modulation :- $V_m = V_c$ then $m_a = 1$

* Hence the envelope of the modulated signal just reaches the zero amplitude axis and shown in figure. This is known as critical modulation

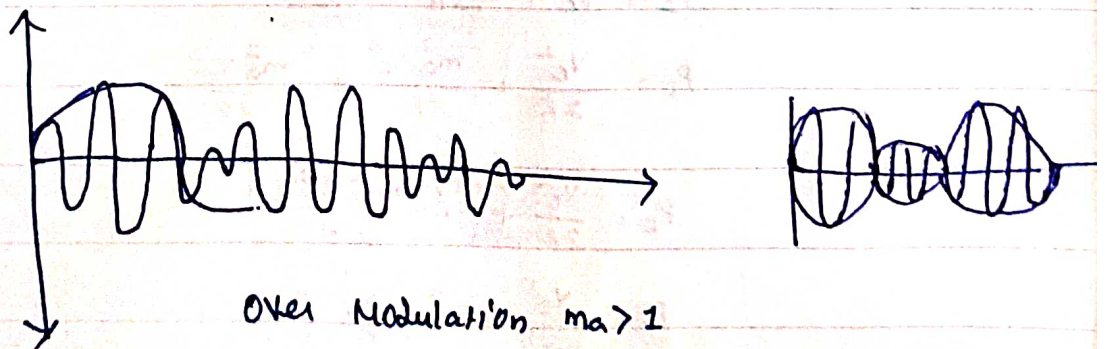


3. Over Modulation :- $V_m > V_c$ then $m_a > 1$

* Amplitude of modulating signal is greater than carrier amplitude. Therefore the portion of envelope of the modulated signal crosses the zero axis.

* So, both positive and negative extensions of modulating signal are cancelled or clipped out.

* The envelopes of message signal are not same. This is called envelope distortion.





Power consume by the amplitude Modulation:-

→ The total Power in Modulated wave will be

$$P_T = P_C + P_{LSB} + P_{USB} \rightarrow (15)$$

$$P_T = \frac{V_{c_{rms}}^2}{R} + \frac{V_{LSB}^2}{R} + \frac{V_{USB}^2}{R} \rightarrow (16)$$

where,

$V_{c_{rms}}$ = RMS value of carrier voltages.

$V_{LSB} = V_{USB}$ = RMS value of upper & lower

side band voltages

where all the 3 voltage rms values ($\frac{1}{\sqrt{2}}$ peak)

and R is the resistance (eg. Antenna resistance) →

In which the power is dissipated the first term of the eq. (16) is the unmodulated carrier power which is given by

R = Resistance in which power is dissipated

$$P_{c_{rms}} = \frac{V_{c_{rms}}^2}{R} = \frac{(V_c / \sqrt{2})^2}{R} = \frac{V_c^2}{2R} \rightarrow (17)$$

$$P_{LSB} = P_{USB} = \frac{V_{LSB}^2}{R} = \frac{\left(\frac{m a V_c}{2}\right)^2}{R} = \frac{m^2 a^2 V_c^2}{8R} \rightarrow (18)$$

V_c = maximum amplitude of carrier wave

$V_{LSB} = \frac{m a V_c}{2}$ = maximum amplitude of side band

Sub (17) & (18) equation in (15)

$$P_T = P_C + P_{LSB} + P_{USB}$$

$$P_T = \frac{V_c^2}{2R} + \frac{m^2 a^2 V_c^2}{8R} + \frac{m^2 a^2 V_c^2}{8R}$$

$$P_T = \frac{V_c^2}{2R} + \left(2 \cdot \frac{m^2 a^2 V_c^2}{8R}\right)$$

$$P_T = \frac{V_c^2}{2R} + \frac{m^2 a^2 V_c^2}{4R}$$

$$P_T = \frac{V_c^2}{2R} \left(1 + \frac{m^2 a^2}{2}\right)$$

We know that

$$P_C = \frac{V_c^2}{2R}$$

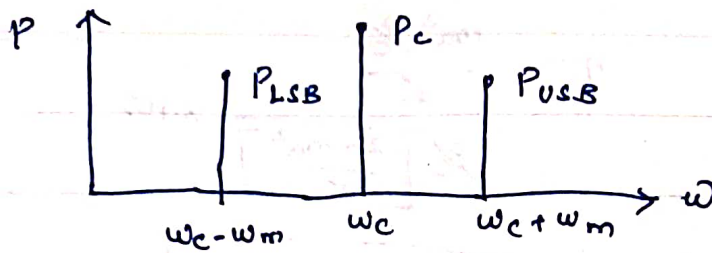


thus, $P_E = P_C \left[1 + \frac{m_a^2}{2} \right]$

$$\frac{P_E}{P_C} = \left[1 + \frac{m_a^2}{2} \right]$$

$m_a = 1$, (or) for 100% modulation

$$\frac{P_E}{P_C} = 1.5 \quad (\text{or}) \quad \boxed{P_E = 1.5 P_C}$$



Efficiency

It can be defined as the ratio of power in sidebands to total power, because side bands only contain the useful information.

$$\% \eta = \frac{\text{Power in side band}}{\text{Total power}} \times 100$$

$$\% \eta = \frac{P_{LSB} + P_{USB}}{P_{total}} \times 100$$

$$= \frac{\frac{m_a^2 \cdot V_c^2}{8R} + \frac{m_a^2 \cdot V_c^2}{8R}}{P_C \left[1 + \frac{m_a^2}{2} \right]} \times 100$$

$$= \frac{\frac{m_a^2 \cdot V_c^2}{8R} + \frac{m_a^2 \cdot V_c^2}{8R}}{P_C \left[1 + \frac{m_a^2}{2} \right]}$$

$$= \frac{2 \left[\frac{m_a^2 \cdot V_c^2}{8R} \right]}{P_C \left[1 + \frac{m_a^2}{2} \right]}$$

$$= \frac{\frac{m_a^2 \cdot V_c^2}{4R}}{P_C \left[1 + \frac{m_a^2}{2} \right]}$$

$$= \frac{\frac{m_a^2 \cdot V_c^2}{4R}}{P_C \left[1 + \frac{m_a^2}{2} \right]} \times 100$$



where

$$P_c = \frac{V_c^2}{2R}$$

$$= \frac{\frac{m_a^2 P_c}{2}}{P_c \left[1 + \frac{m_a^2}{2} \right]} \times 100$$

$$\eta = \frac{\frac{m_a^2 P_c}{2}}{P_c \left[\frac{2+m_a^2}{2} \right]} \times 100$$

$$\boxed{\eta \% = \frac{m_a^2}{2+m_a^2} \times 100}$$

If $m_a = 1$

then

$$\eta \% = \frac{1^2}{2+1^2} \times 100$$

$$= \frac{1}{3} \times 100$$

$$= 33.3\%$$

* From this, we conclude that the efficiency of the Power Consumption is 33.3%

* It is concluded that only 33.3% of Power is used for transmission and the remaining Power is wasted in the carrier transmission along with the sidebands.



Double side band suppressed carrier AM (DSB-SC-AM)

* Two important Parameters of a communication system.

(i) Transmitting Power

(ii) Bandwidth

* Hence saving of power & bandwidth are highly desirable in a communication system.

* AM with carrier scheme, there is wastage in both transmitted power & bandwidth

* In order to save the power in amplitude modulation the carrier may be suppressed, because it does not contain any useful information.

* This scheme is called as the Double side Band suppressed carrier Amplitude Modulation (DSB-SC-AM).

* It contains only LSB & USB terms, resulting that a transmission bandwidth is twice the frequency of the message signal

Let,

$$\text{Modulating signal } V_m(t) = V_m \sin \omega_m t$$

$$\text{carrier signal } V_c(t) = V_c \sin \omega_c t$$

When, multiplying both carrier & message signal,

$$\begin{aligned} V(t)_{\text{DSB-SC}} &= V_m(t) * V_c(t) \\ &= V_m \sin \omega_m t * V_c \sin \omega_c t \\ &= V_m \cdot V_c \cdot \sin \omega_m t \cdot \sin \omega_c t \end{aligned}$$

We know,

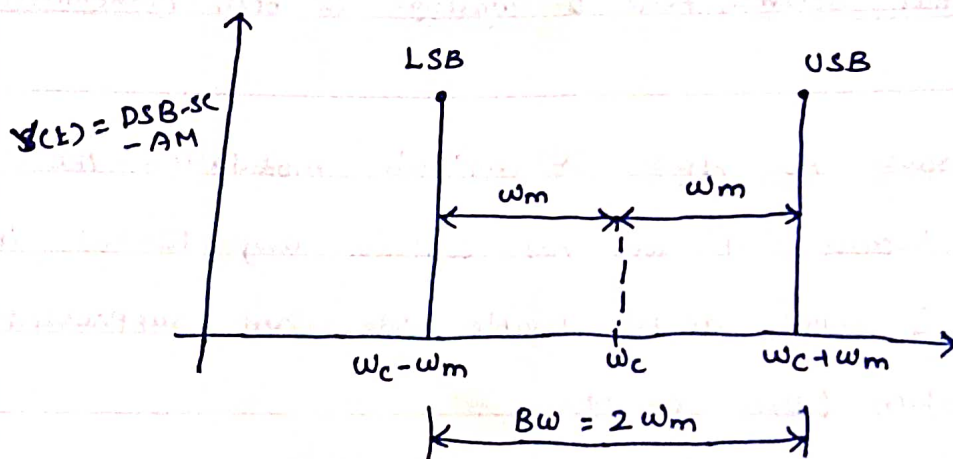
$$\sin \omega_m t \cdot \sin \omega_c t = \frac{\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t}{2}$$

$$= \frac{V_m V_c}{2} [\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t] \rightarrow \textcircled{1}$$



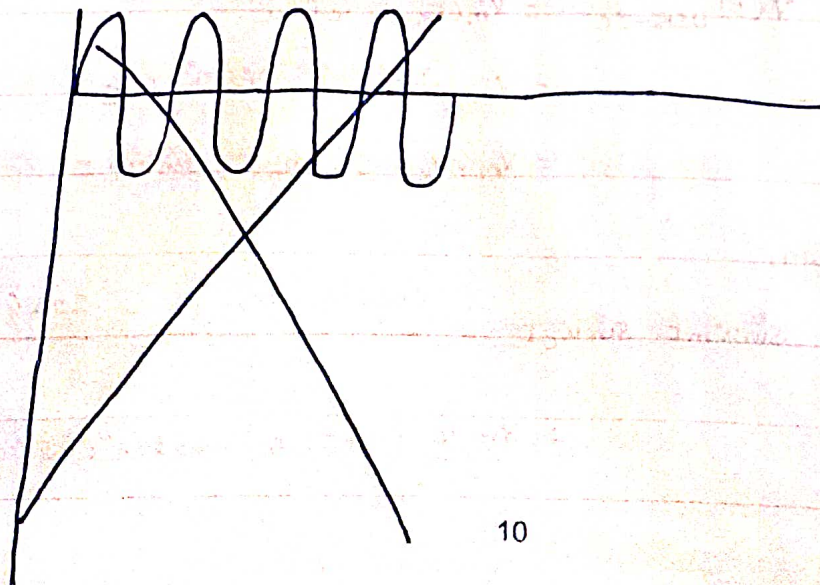
* In this case, the product of $V_m(t)$ & $V_c(t)$ produces the DSB-SC-AM signal for this it requires product modulator & generator.

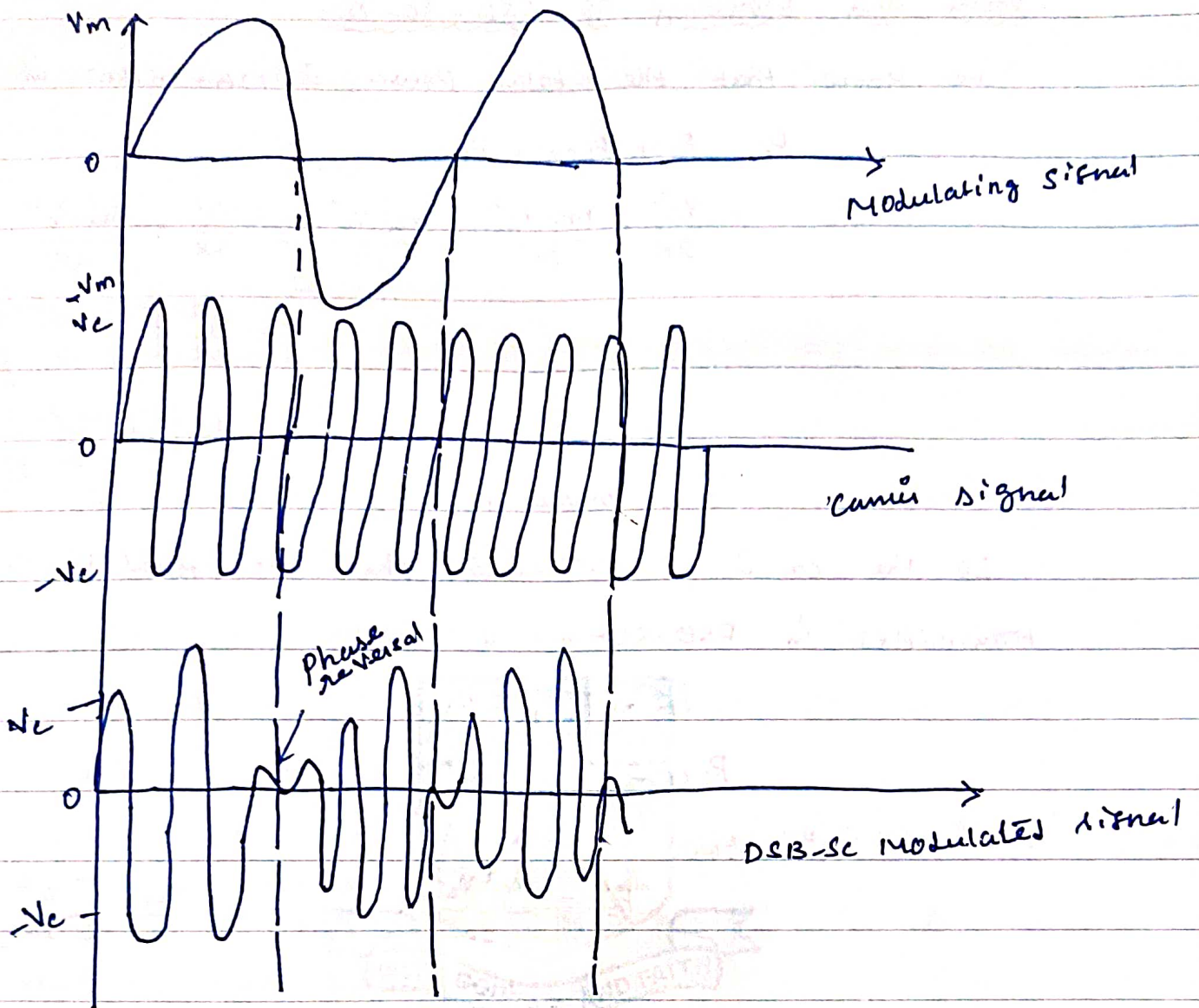
* The above equ-① represents the DSB-SC signal carrier term $V_c \sin \omega_c t$ is missing and only two sidebands are present.



Frequency spectrum of DSB-SC-AM

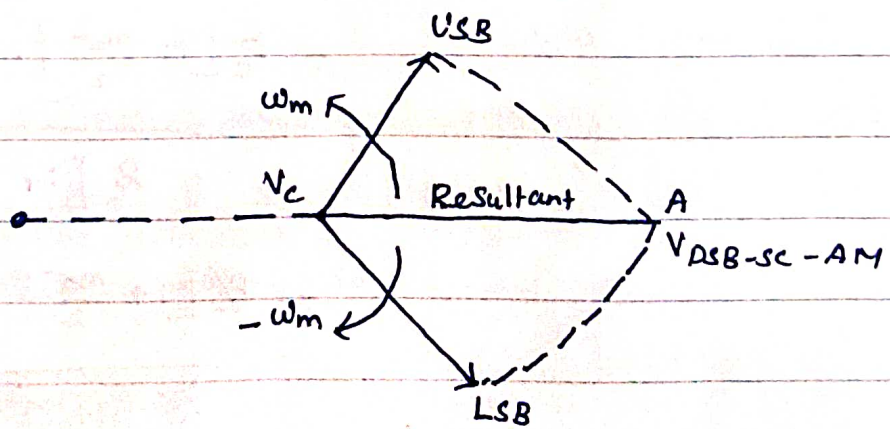
From the above figure carrier term ω_c is suppressed. It contains only two sideband terms having the frequency of $(\omega_c + \omega_m)$ and $(\omega_c - \omega_m)$. Hence this scheme is known as DSB-SC-AM.





Graphical Representation of DSB-SC-AM

Phasor Diagram of DSB-SC-AM





Power and Efficiency of DSB-SC-AM

We know that the total power is transmitted in AM

$$P_T = P_C + P_{LSB} + P_{USB}$$

$$= \frac{V_c^2}{2R} + \frac{m_a^2 V_c^2}{8R} + \frac{m_a^2 V_c^2}{8R} = \frac{V_c^2}{2R} + \frac{m_a^2 V_c^2}{4R}$$

$$= \frac{V_c^2}{2R} \left[1 + \frac{m_a^2}{2} \right]$$

$$= P_C \left[1 + \frac{m_a^2}{2} \right]$$

$$\text{where } P_C = \frac{V_c^2}{2R}$$

If the carrier is suppressed, then the total power transmitted in DSB-SC-AM is

$$P'_T = P_{LSB} + P_{USB}$$

$$P_{LSB} = P_{USB} = \frac{m_a^2 V_c^2}{8R}$$

therefore,

$$P'_T = \frac{m_a^2 V_c^2}{8R} + \frac{m_a^2 V_c^2}{8R} = \frac{m_a^2}{2} \left[\frac{V_c^2}{2R} \right]$$

$$P'_T = \frac{m_a^2}{2} P_C$$

$$\text{Power saving} = \frac{P_T - P'_T}{P_T}$$

$$= \frac{P_C \left[1 + \frac{m_a^2}{2} \right] - P_C \left[\frac{m_a^2}{2} \right]}{P_C \left[1 + \frac{m_a^2}{2} \right]}$$

$$= \frac{P_C \left[1 + \frac{m_a^2}{2} - \frac{m_a^2}{2} \right]}{P_C \left[1 + \frac{m_a^2}{2} \right]}$$

$$= \frac{P_C \left[1 + \frac{m_a^2}{2} - \frac{m_a^2}{2} \right]}{P_C \left[1 + \frac{m_a^2}{2} \right]}$$

$$= \frac{2 + m_a^2 - m_a^2}{2} = \frac{1}{\left[1 + \frac{m_a^2}{2} \right]}$$

$$= \frac{1}{\left[1 + \frac{m_a^2}{2} \right]}$$

$$= \frac{1}{\left[1 + \frac{m_a^2}{2} \right]}$$



$$\text{Power Saving} = \frac{2}{2+m_a^2} \times 100$$

If $m_a = 1$ then power saving

$$= \frac{2}{3} \times 100 = 66.7\%$$

therefore, 66.7% of power is saved. Due to the suppression of the carrier wave, the power saving is increasing from 33.3% to 66.7%.

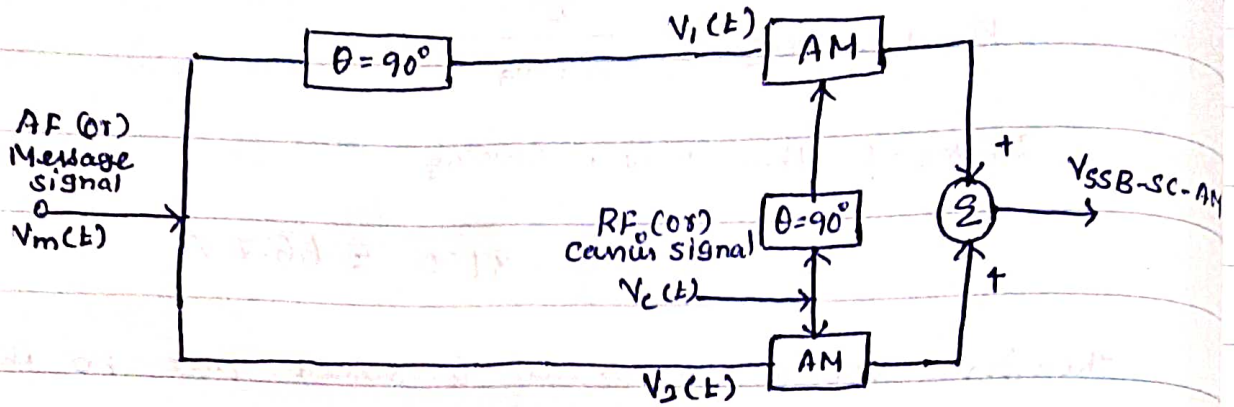
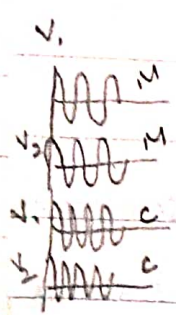
⇒ Single Side Band Suppressed Carrier AM (SSB-SC-AM)

* In AM with carrier both the transmitting power and bandwidth are wasted.

* Hence, the DSB-SC-AM scheme has been introduced in which power is saved by suppressing the carrier component but the bandwidth remains the same (i.e. $B.W = 2W_m$).

* Further increase in the saving of power is possible by eliminating one side band in addition to the carrier component, because the USB and LSB are uniquely related by symmetry about the carrier freq.

* In addition to that, transmission bandwidth can be reduced into half i.e. one side band is suppressed along with the carrier. This scheme is known as SSB-SC-AM.



Block diagram of SSB-SC-AM

* In order to suppress one of the side band

- Input signal fed to the Modulator '1' is 90° out of phase with that of the signal fed to the Modulator '2'

- Let, $V_1(t) = V_m \cdot \sin(\omega_m t + 90^\circ) \cdot V_c \sin(\omega_c t + 90^\circ)$

$$V_1(t) = V_m \cos \omega_m t \cdot V_c \cos \omega_c t$$

$$V_2(t) = V_m \cdot \sin \omega_m t \cdot V_c \sin \omega_c t$$

$$\therefore V(t)_{SSB} = V_1(t) + V_2(t)$$

$$= V_m \cos \omega_m t \cdot V_c \cos \omega_c t + V_m \sin \omega_m t \cdot V_c \sin \omega_c t$$

$$= V_m V_c [\cos \omega_m t \cdot \cos \omega_c t] + V_m V_c [\sin \omega_m t \cdot \sin \omega_c t]$$

$$= V_m V_c [\sin \omega_m t \cdot \sin \omega_c t + \cos \omega_m t \cdot \cos \omega_c t]$$

we know that,

$$\sin A \sin B + \cos A \cos B = \frac{\cos(A-B)}{2}$$

$$\text{Hence, } V(t)_{SSB} = \frac{V_m V_c}{2} [\cos(\omega_c - \omega_m)t] \rightarrow (1)$$

we know that for DSB-SC-AM,

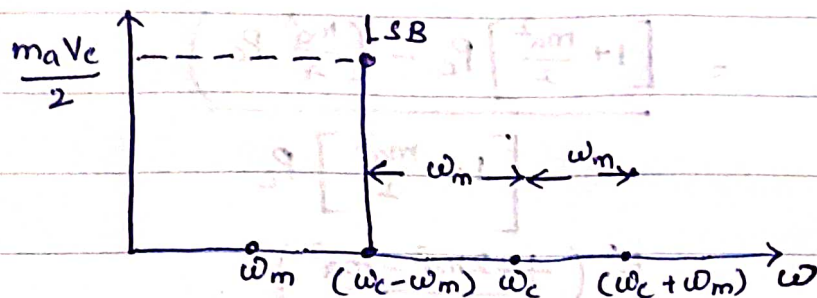
$$V_{DSB}(t) = \frac{V_m V_c}{2} [\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t]$$

when comparing equations (1) & (2) one of the side band is suppressed. Hence this scheme is known as SSB-SC-AM



$$\frac{m_a^2 V_c^2}{4} = \frac{m_a^2}{4} \times \frac{V_c^2}{2R} = \frac{m_a^2}{4} P_c$$

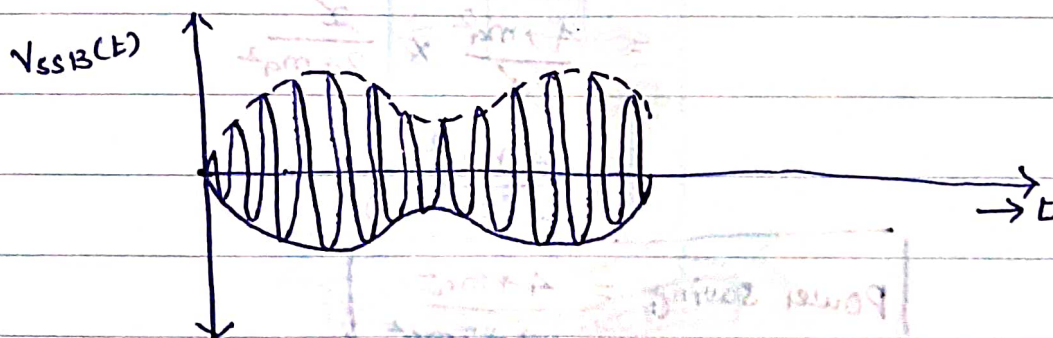
Frequency Spectrum of SSB-SC-AM



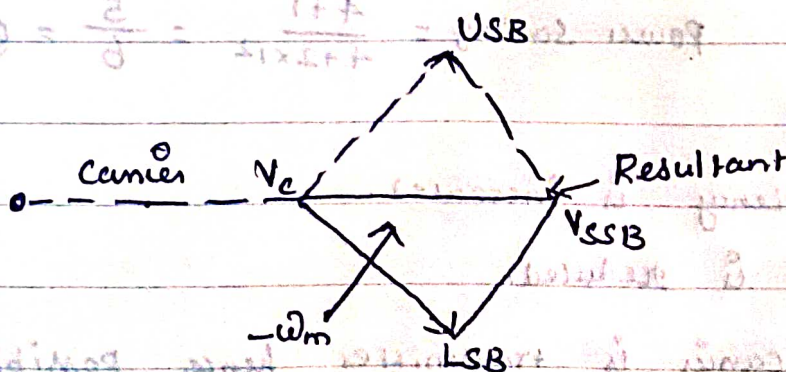
Therefore, Bandwidth is reduced to $2\omega_m$ to ω_m

$$BW = \omega_m$$

Graphical Representation :-



Phasor Diagram of SSB-SC-AM :-



Power calculation :-

$$\text{Power in SSB-SC-AM is } P_L = P_{LSB} = \frac{m_a^2 P_c}{4}$$

where

$$P_{LSB} = \frac{m_a^2 V_c^2}{4R} = \frac{m_a^2}{4} P_c$$

$$(i.e) P_c = \frac{V_c^2}{2R}$$



$$\begin{aligned}\text{Power Saving} &= \frac{P_E - P_E''}{P_E} \\ &= \frac{\left[1 + \frac{ma^2}{2}\right] P_C - \left(\frac{ma^2}{4} P_C\right)}{\left[1 + \frac{ma^2}{2}\right] P_C} \\ &= \frac{P_C \left(\frac{4 + 2ma^2 - ma^2}{4}\right)}{P_C \left(1 + \frac{ma^2}{2}\right)} \\ &= \frac{\frac{4 + ma^2}{4}}{\frac{2 + ma^2}{2}} \\ &= \frac{4 + ma^2}{4} \times \frac{2}{2 + ma^2} \\ &= \frac{4 + ma^2}{2(2 + ma^2)}\end{aligned}$$

$$\boxed{\text{Power saving} = \frac{4 + ma^2}{4 + 2ma^2}}$$

If $ma = 1$

$$\text{Power saving} = \frac{4 + 1^2}{4 + 2 \times 1^2} = \frac{5}{6} = 83.3\%$$

Advantages

- * Efficiency is increased.
- * B.W is reduced.
- * No carrier is transmitted hence possibility of interference with other channels are avoided.
- * Improved signal to noise ratio.



Dis Advantages:-

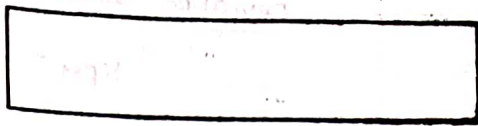
* Major drawback is that the transmission and reception of SSB become more complex and the required Performance Standard is Very high.

Application:-

- * Police wireless communication
- * SSB telegram system
- * Point to point radio telephone communication
- * VHF & UHF communication system.


 ADDITIONAL BOOK
 UNIT-3

SIGNATURE OF HALL INVIGILATOR

ANGLE MODULATION:-

* Angle Modulation is the process by which the angle (i.e. Freq (Or) Phase) of the carrier signal is changed in accordance with the instantaneous amplitude of modulating (or) message signal.

* As per the definition of angle modulation, the phase angle of carrier signal is varied in accordance with the instantaneous amplitude of message signal.

* we know $\phi(t) = \omega_c t + \theta(t)$

(i.e) whenever the frequency of carrier is varied. Thus FM & PM even if either of the terms are varied.

(i) Instantaneous Phase (ϕ)

It can be defined as the Phase of the carrier at any instant of time.

(ii) Instantaneous Phase deviation ($\theta(t)$)

It is defined as the change in phase of the carrier at any instant of time. with respect to its reference phase.

(iii) Instantaneous Frequency (ω_i)

It is the frequency of the carrier at any instant of time.

(i.e)

$$\omega_i(t) = \frac{d}{dt} \phi(t) = \frac{d}{dt} [\omega_c t + \theta(t)] = \omega_c + \theta'(t)$$

(iv) Instantaneous frequency deviation $\phi'(t)$

It is the change in frequency of the carrier. It can be defined as the first time derivation of instantaneous phase deviation.

Deviation sensitivity

$$K_{FM} = \frac{\Delta \omega}{V_m}$$

= $\frac{\text{change in output frequency}}{\text{change in input voltage}}$.

|| PM

$$K_{PM} = \frac{\Delta \theta}{V_m}$$

$$\text{or } \Delta \theta = K_{PM} V_m$$

FREQUENCY MODULATION

* Frequency Modulation can be defined as the process by which the frequency of the carrier wave is altered in accordance with the instantaneous amplitude of modulating (or) message signal.

* The mathematical representation,

$$\text{message signal } V_m(t) = V_m \cos \omega_m t \rightarrow (1)$$

$$\text{carrier signal } V_c(t) = V_c \sin [\omega_c t + \theta] \rightarrow (2)$$

where

$V_m \rightarrow$ max amplitude of message signal

$V_c \rightarrow$ max amplitude of carrier signal

$\omega_m \rightarrow$ angular frequency of modulating signal

$\omega_c \rightarrow$ angular frequency of carrier signal

$\theta \rightarrow$ Total instantaneous phase angle of carrier

$$\theta = (\omega_c t + \theta)$$

$$V_c(t) = V_c \sin \phi = V_c \sin (\omega_c t + \theta) \rightarrow (3)$$

* To find angular velocity, diff. the equation (3) w.r.t. 't'

$$(i) \frac{d\phi}{dt} = \omega_c = \phi'(t)$$

* Frequency Modulation \rightarrow Frequency of carrier signal is changed in accordance with message signal

$$\omega_i = \omega_c + k V_m(t) = \omega_c + k V_m \cos \omega_m t \rightarrow (4)$$

where

$k =$ constant of proportionality.

* To find instantaneous phase angle, integrate equation (4)

$$\phi_i = \int \omega_i dt = \int (\omega_c + k V_m \cos \omega_m t) dt$$
$$= \omega_c t + \frac{k V_m}{\omega_m} \sin \omega_m t + \theta,$$

$\theta =$ Integration constant, it is neglected

* The instantaneous amplitude of the modulating signal is given by,

$$V(t)_{FM} = V_c \sin \phi_i = V_c \sin \left(\omega_c t + \frac{k V_m}{\omega_m} \sin \omega_m t \right) \rightarrow (5)$$

$$V(t)_{FM} = V_c \sin (\omega_c t + m_f \sin \omega_m t) \rightarrow (6)$$

where

$$m_f = \frac{k_{FM} V_m}{\omega_m}$$

modulation index of FM

From equation (4) the instantaneous angular frequency of FM signal

$$\omega_i = \omega_c + kV_m \cos \omega_m t$$

Note

The maximum and minimum value of cosine term is ± 1

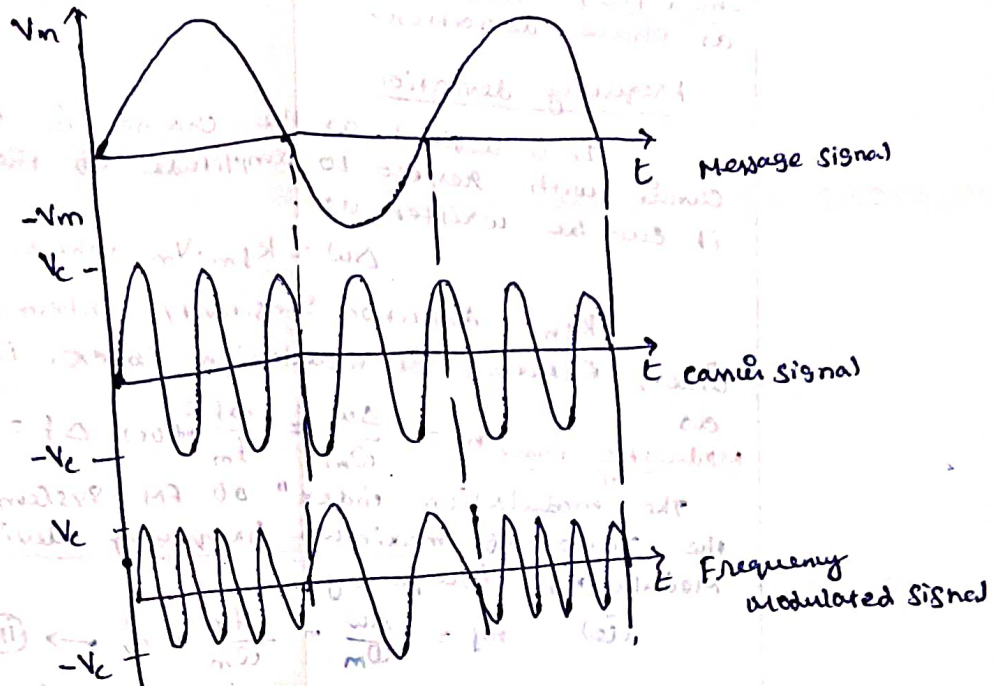
Hence

Maximum value of angular frequency $\omega_{max} = \omega_c + kV_m$

Minimum value " " " $\omega_{min} = \omega_c - kV_m$

* Frequency deviation is given by

$$\omega_d = \omega_{max} - \omega_c = \omega_c - \omega_{min} = kV_m \rightarrow (7)$$



PHASE MODULATION:-

Phase modulation can be defined as the process by which changing the phase of the carrier signal in accordance with the instantaneous amplitude of the message signal. Amp, Frequency remains constant.

Let Modulating signal is given by $V_m(t) = V_m \cos \omega_m t \rightarrow (1)$

The carrier signal $V_c(t) = V_c \sin(\omega_c t + \theta) \rightarrow (2)$

where

θ = phase angle of carrier signal. It is changed in accordance with the amplitude of the message signal $V_m(t)$

$$\theta = k_{PM} V_m(t) = k_{PM} V_m \cos \omega_m t \rightarrow (8)$$

where

k_{PM} = phase deviation sensitivity

After phase modulation the instantaneous voltage

$$V_{pm}(t) = V_c \sin(\omega_c t + \theta)$$

$$= V_c \sin(\omega_c t + k_{PM} V_m \cos \omega_m t) \rightarrow (9)$$

$$V_{pm}(t) = V_c \sin(\omega_c t + m_p \cos \omega_m t) \rightarrow (10)$$

where

$$m_p = k_{PM} V_m$$

Modulation index of phase modulation.

Phase Deviation and Modulation Index

Phase Deviation:-

The eq (6) compared with equation (3) we get

$$V(t)_{FM} = V_c \sin(\omega_c t + m_f \sin \omega_m t) \rightarrow (6)$$

$$V_c(t) = V_c \sin \phi = V_c \sin(\omega_c t + \theta) \rightarrow (3)$$

$$V_{fm}(t) = V_c \sin(\omega_c t + m_f \sin \omega_m t) = V_c \sin[\omega_c t + \theta(t)]$$

where $\theta(t)$ = instantaneous phase deviation = $m_f \sin \omega_m t$

If the modulating signal is single tone (or) sinusoid, then the phase angle of the carrier varies from its un-modulated signal during the modulation process is known as phase deviation.

Frequency Deviation:-

It is defined as the change in frequency of the carrier with respect to amplitude of the modulating signal, it can be written as

$$\Delta \omega = k_{fm} \cdot V_m \text{ where}$$

k_{fm} = deviation sensitivity, in terms of modulation index, ~~it can be written as~~ it can be written as

$$\text{Modulation index } m = \frac{\Delta \omega}{\omega_m} = \frac{\Delta f}{f_m} \text{ (or) } \Delta f = m f_m$$

The "modulation index" of FM system can be defined as the ratio of maximum frequency deviation to the modulating frequency.

$$(e) \quad m_f = \frac{\Delta \omega}{\omega_m} = \frac{k_f V_m}{\omega_m} = \delta \rightarrow (ii)$$

$$\Delta \omega = k_f V_m \rightarrow \text{maximum frequency deviation}$$

For PM

The modulation index depends on the modulating signal (e) $m_p = k_{pm} V_m$ where k_{pm} = deviation sensitivity

Different types of FM

1) Narrow Band F.M

We know that the bandwidth of an FM signal depends upon the frequency deviation ($\Delta \omega = k_f \cdot x(t)$). If frequency deviation is low it means k_f is low then narrow band FM is formed.

2) Wideband FM

If frequency deviation ($\Delta \omega = k_f \cdot x(t)$) is high, it means frequency sensitivity k_f is high result as bandwidth will be wide hence wide band is formed.

Comparison of WBFM and NBFM

WBFM	NBFM
(i) Modulation index is greater than 1 and minimum value = 5	Modulation index is less than 1
(ii) Frequency deviation = 75 kHz	Frequency deviation 5 kHz
(iii) Modulating frequency range from 30 Hz - 15 kHz	Modulating frequency = 3 kHz
(iv) Band width 15 times NBFM	Bandwidth = 2 fm
(v) Noise is more suppressed	Less suppressing of noise
(vi) use:- Entertainment and broadcasting	use:- Mobile communication

Generation of narrow band FM

* Narrow band FM for which modulation index is small compared to one radian.

* wide band FM for which modulation index is large compared to one radian.

Let the message signal be represented as

$$V_m(t) = V_m \cos \omega_m t$$

Let the carrier signal be given by

$$V_c(t) = V_c \sin(\omega_c t + \theta) = V_c \sin \phi$$

where

$$\phi = (\omega_c t + \theta) \rightarrow (1)$$

Diff. eqn No (1) w.r.t 't' we get

$$\frac{d\phi}{dt} = \omega_c \Rightarrow \text{Angular frequency of carrier signal}$$

After frequency modulation

$$\omega_i = \omega_c + k V_m(t) = \omega_c + k V_m \cos \omega_m t \rightarrow (2)$$

The frequency deviation is maximum when

$$\cos \omega_m t = \pm 1 \text{ hence } \omega_i = \omega_c \pm k V_m$$

The frequency deviation is proportional to the amplitude of modulating voltage, hence it can be written as

$$2\pi \Delta f = k V_m$$

$$\omega_i = \omega_c \pm 2\pi \Delta f \cdot \cos \omega_m t$$

$$\phi_i = \int \omega_i dt = \int (\omega_c + 2\pi \Delta f \cdot \cos \omega_m t) dt$$

$$\phi_i = \omega_c t + \frac{\Delta f}{f_m} \sin \omega_m t$$

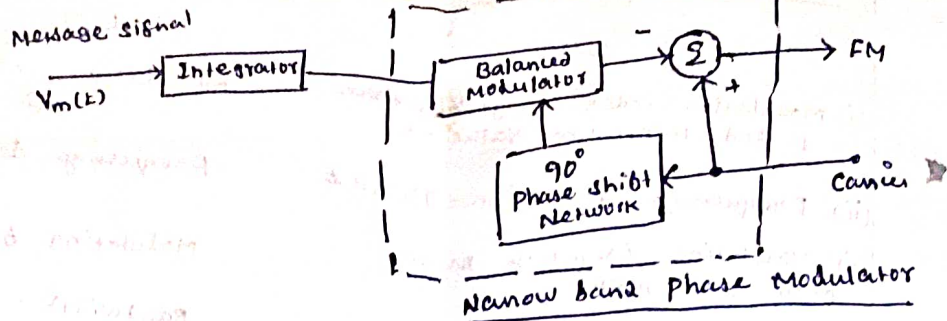
$$V_{fm}(t) = V_c \cdot \sin \phi_i t$$

$$= V_c \sin \left(\omega_c t + \frac{\Delta f}{f_m} \sin \omega_m t \right)$$

$$= V_c \sin \left(\omega_c t \pm m_f \sin \omega_m t \right) \Rightarrow \sin(A+B)$$

$$V_{fm}(t) = V_c \sin \omega_c t \cdot \cos(m_f \sin \omega_m t) + V_c \cos \omega_c t \cdot \sin(m_f \sin \omega_m t)$$

Note



- * This modulator involves splitting the carrier wave into two paths
 - ✓ → one path is direct
 - ✓ → other path contains a -90° phase shift N/W and a Product Modulator.

→ combination of which generates DSB-SC-AM signal.
(Double-sideband suppressed-carrier)

→ The difference b/w these two signals produce narrow band FM with some distortion.

- * Ideally FM signal has a constant envelope, but the modulating signal produced by narrowband FM

- * Ideal condition in two fundamental respects

- 1) Envelope contains a residual amplitude modulation and varies with time.
- 2) It produces some harmonic distortions.

GENERATION OF WIDE BAND FM

wide band FM for which modulation index is large
compared to narrow band AM.

Let the message signal be represented as

$$V_m(t) = V_m \cos \omega_m t$$

Let the carrier signal be represented by

$$V_c(t) = V_c \sin(\omega_c t + \theta) = V_c \sin \phi$$

$$\text{where } \phi = (\omega_c t + \theta) \rightarrow \text{①}$$

Diff. equation ① w.r.t 't'

$$\frac{d\phi}{dt} = \omega_c \rightarrow \text{Angular frequency of the carrier signal.}$$

Angular frequency Modulation

$$\omega_i = \omega_c + K V_m(t) = \omega_c + K V_m \cos \omega_m t$$

The frequency deviation is maximum when $\cos \omega_m t = \pm 1$

$$\therefore \omega_i = \omega_c \pm K V_m$$

and the frequency deviation is proportional to the amplitude of the modulation voltage.

$$\text{Hence } 2\pi \Delta f = K V_m$$

$$\omega_i = \omega_c \pm 2\pi \Delta f \cdot \cos \omega_m t$$

$$\phi_i = \int \omega_i dt = \int (\omega_c + 2\pi \Delta f \cdot \cos \omega_m t) dt$$

$$\phi_i = \omega_c t + \frac{\Delta f}{f_m} \sin \omega_m t$$

$$V_{fm}(t) = V_c \sin \phi_i$$

$$= V_c \sin \left(\omega_c t + \frac{\Delta f}{f_m} \sin \omega_m t \right)$$

$$= V_c \sin (\omega_c t + m_f \sin \omega_m t)$$

$$V_{fm}(t) = V_c \sin \omega_c t \cdot \cos (m_f \sin \omega_m t) + V_c \cos \omega_c t \cdot \sin (m_f \sin \omega_m t)$$

we know that

$$V_{fm}(t) = V_c \sin(\omega_c t + m_f \sin \omega_m t)$$

The eq (2) rewritten by using Exponential form:-

$$V_{fm}(t) = V_c e^{j(\omega_c t + m_f \sin \omega_m t)} = \text{Re} [V_c e^{j(\omega_c t + m_f \sin \omega_m t)}]$$

when $\tilde{V}(t) = V_c [e^{j m_f \sin \omega_m t}] \rightarrow (3)$

eq (3) could be expanded by Fourier series

$$V_{fm}(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_m t}$$

where $C_n =$ Fourier co-efficient

$$C_n = \frac{1}{T_m} \int_{-T_m/2}^{T_m/2} \tilde{V}(t) e^{-jn\omega_m t} dt$$

FM TRANSMITTERS

* The frequency modulated wave can be produced by two methods namely,

- (1) Directly modulated FM transmitter.
- (2) Indirectly modulated FM transmitter.

* In a directly modulated FM transmitter the modulating system directly produce FM waves by varying the master oscillator frequency.

* Alternately, indirectly modulated FM transmitter generates the Phase Modulated signals. The PM wave is then converted into frequency modulated wave.

* The basic difference b/w the two circuits is that,

- First circuit employs an LC circuit in Master oscillator.
(Change with changes in circuit parameters)

- Second circuit uses the crystal oscillator as a master oscillator,

* First ~~frequency~~ circuit employs some form of AFC, and it produces more frequency deviation and requires less number of frequency multipliers.
(Automatic freq control)

* Phase modulator circuit produces smaller frequency deviation and requires more number of frequency multiplier stages.

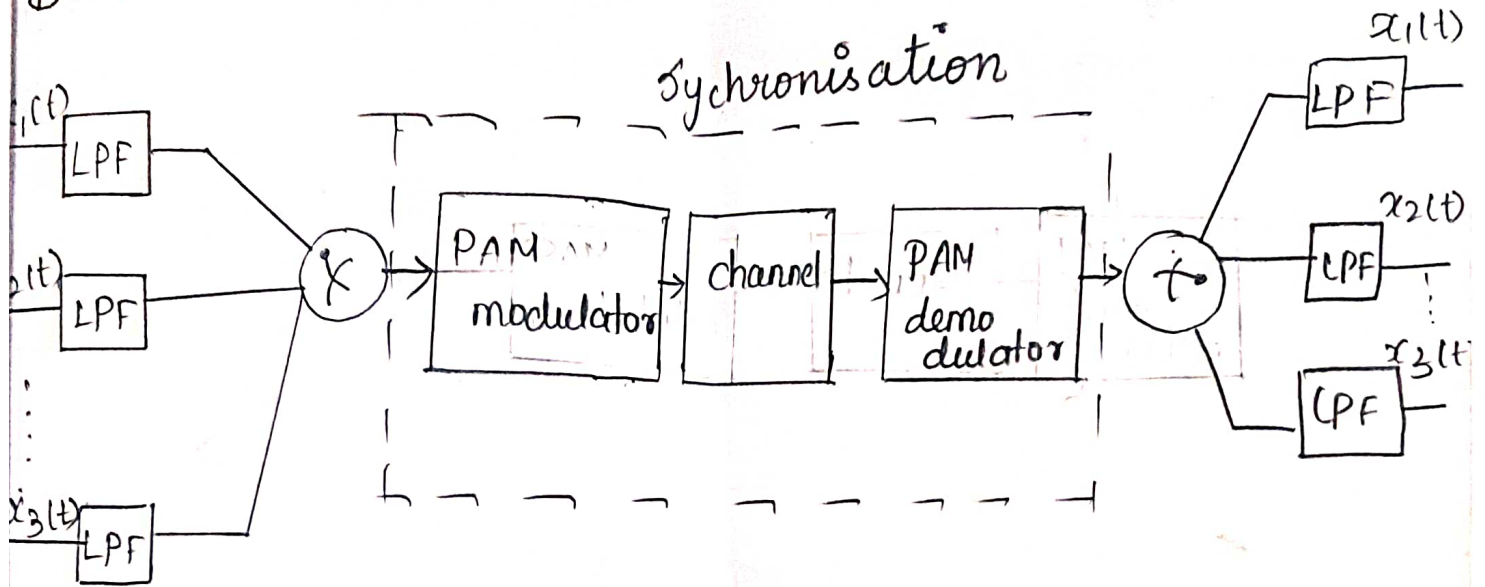
I Direct FM transmitter:-

(Frequency modulated transmitter using reactance tube modulators)

Time division multiplexing - PAM

Multiplexing refers to converting many signal to one signal. Here Time division multiplexing refers to converting many signals to one signal with respect to time.

BLOCK DIAGRAM OF TDM-PAM

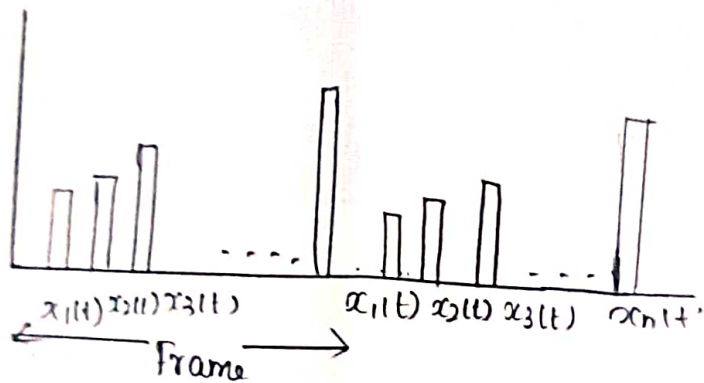


The signals $x_1(t)$, $x_2(t)$, ... $x_n(t)$ are the input signals. The signals are given to low pass filter to remove anti-aliasing effect. Then the signals are i.e. filtered signals are given to Commutator switch. In Transmitter side commutator switch rotates in anticlockwise direction. In the receiver side commutator switch rotates in clockwise direction.

After given in the switch the signals are passed through the channel. In the channel

The signal gets modulated and demodulated.
Both the Commutator switch gets synchronised.
Then the demodulated signal is given to receiver
Commutator switch. Again it is low pass filtered
Finally we get the signal.

One complete rotation of the switch is
called frame:



Date :

Pulse Time Modulation (PTM)

In pulse time modulation, the timing characteristics such as width, position or frequency are varied with the amplitude of the input signal.

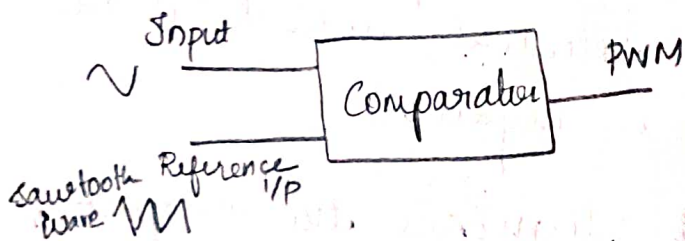
PTM signals are of two types.

1. Pulse Width Modulation (PWM)
2. Pulse position Modulation (PPM).

Pulse Width Modulation (PWM)

In PWM the width of the pulse is varied in accordance with the amplitude of input signal.

Generation :



PWM signal can be generated by using a comparator. Modulating and sawtooth signals forms the input of comparator.

It is one of the simplest method of generation of PWM signal.

One input of the comparator is message signal (sine wave) and other be the sawtooth



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Signal which is the reference input.

Considering both positive and negative sides, the maximum of the input signal should be less than that of sawtooth signal.

The comparator will compare the two signals to generate PWM at output.

When sawtooth is at minimum value which is less than the minimum of the input signal, then positive input of the comparator is at higher potential which gives the comparator o/p as positive.

When sawtooth signal rises and is at the maximum value, the negative input of the comparator is at higher potential, which will produce the output to be negative.

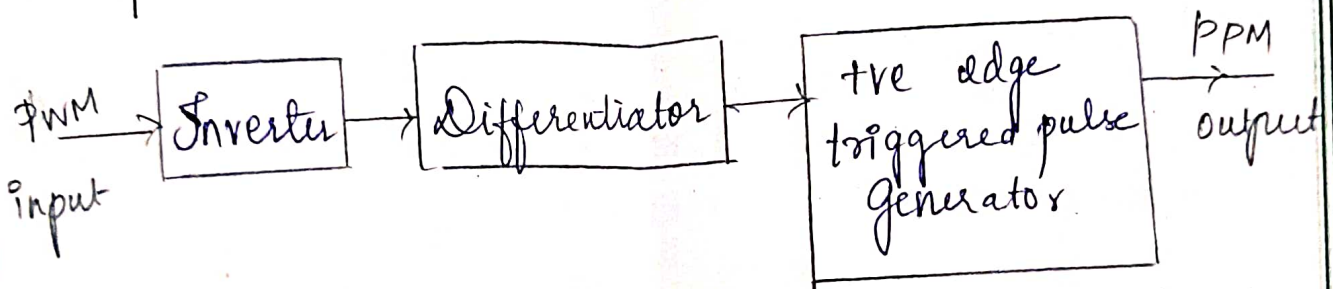
Magnitude of input determines the output, i.e., width of the pulse generated signal is directly proportional to the amplitude of modulating signal.

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Pulse Position Modulation (PPM).

In PPM the amplitude and width of the pulses is kept constant but the position of each pulse is varied in accordance with the amplitude of the message or modulating signal.

Generation



The PPM signal is generated with PWM. The PWM signal is sent to an Inverter which reverses the polarity of the pulse.

This is then followed by a differentiator which generates positive spikes for PWM signal from high to low and negative spikes for low to high transition.

These spikes are then fed to the positive edge triggered pulse generator which generates fixed width pulses when a positive spike appears coinciding with the falling edge of PWM.

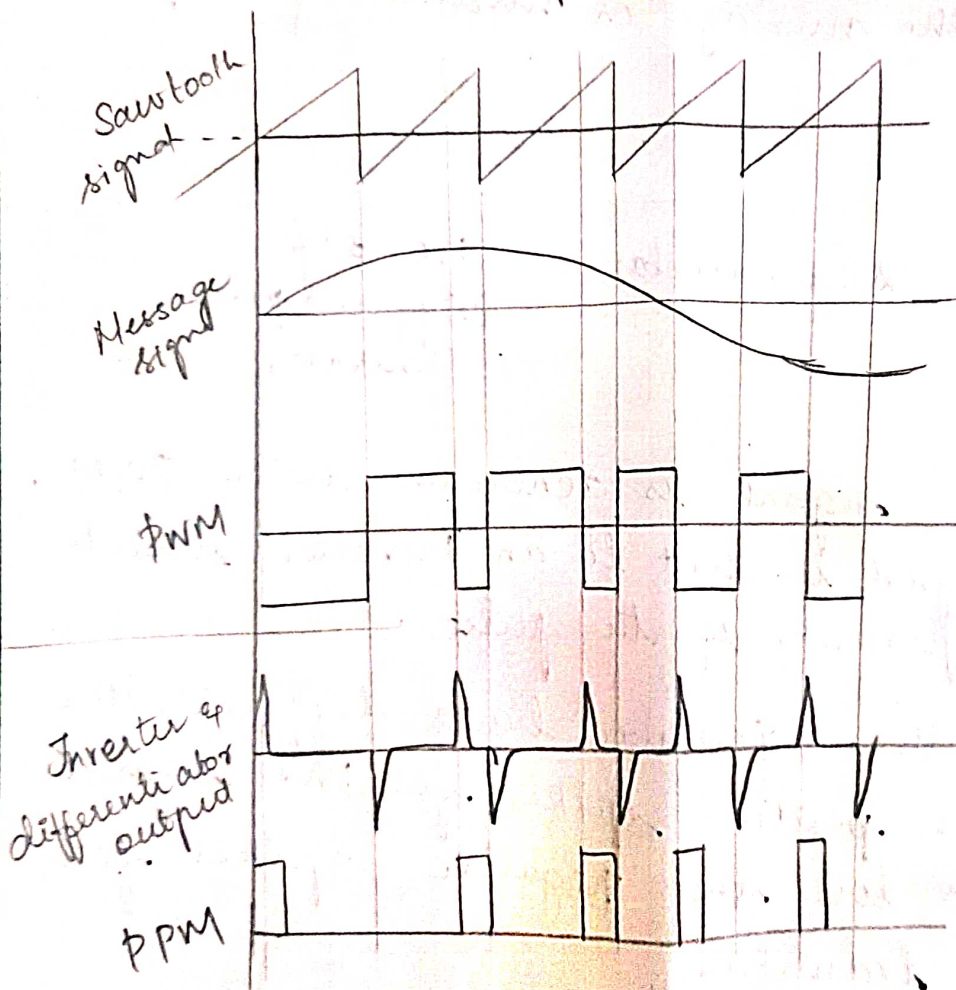


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Thus PPM is generated at the output which is shown in figure.

Generation of PWM and PPM



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Detection of PWM and PPM :

For PWM demodulation or detection a ramp is added at the positive edge which will stop at the arrival of a negative edge. The ramp will attain different heights attained are directly proportional to the pulse width and in turn the amplitude of the message signal. This is then passed through a LPF (Low pass filter) where it will follow the envelope that is the message signal, which produces the demodulated signal at the output.

For PPM demodulation, ramp is used which starts at the positive edge of the one pulse and stops at the positive edge of the next pulse. Thus the height of the generated ramp is determined by the delay between the pulses which indirectly follows the amplitude of the modulating signal. This is then passed through Low pass filter which filters the envelope information as the demodulated signal.



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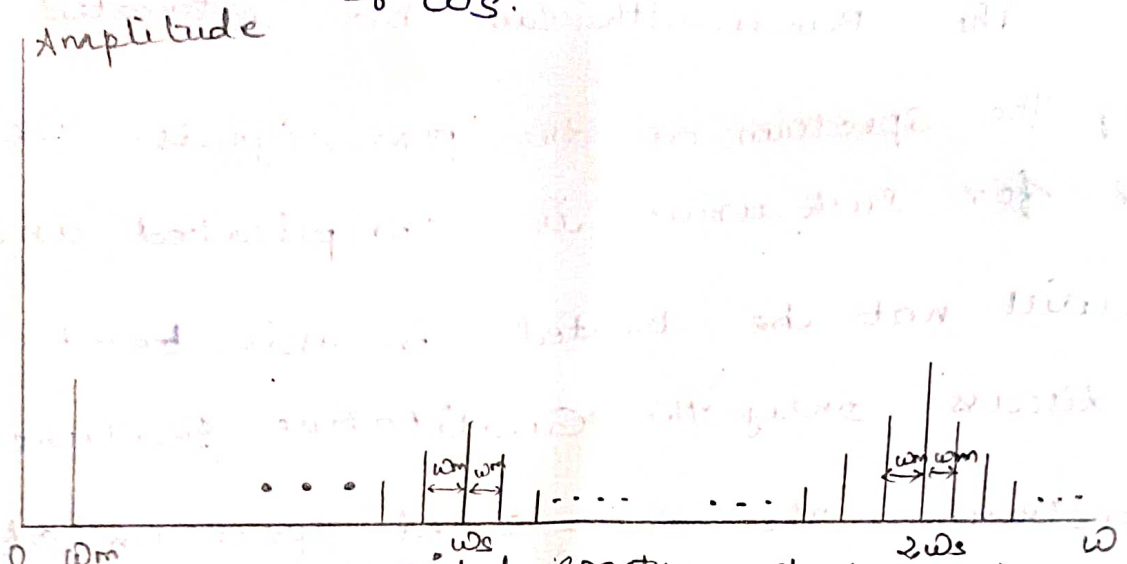
bandwidth of PWM signals :

The Bandwidth can be estimated by observing the spectrum of the PWM signals. The spectral for such waves is complicated and hence will not be treated in this text. We will discuss only the Qualitative features of the spectrum. The one-sided spectrum of a PWM signal is shown in figure. Assume that the modulating signal is a single-tone sinusoidal of the frequency ω_m , and the sampling frequency is ω_s . The PWM spectrum has the following frequency ω_m , components.

- i, A d.c component at $\omega = 0$, which represents the average value of the pulses.
- ii, The modulating frequency ω_m ,
- iii, The harmonics of the sampling frequency ω_s .

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iv, sidebands space by ω_m , centered around each harmonic of ω_s .



The presence of harmonics of ω_s is due to

the contribution of the unmodulated pulse train which may be taken as the carrier of the PWM wave. Each harmonic of ω_s is associated with the sidebands of an FM type. The sidebands of each ω_s extend to infinity outward, but with a decaying magnitude. However, the useful message band is available in a band $0 - \omega_m$ and hence a low pass filter can be used to recover the message from the PWM. But the output of LPF

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is distorted due to the presence of cross-modulated terms that lie in the baseband. The lower sidebands of ω_c may extend lie in the message baseband to cause distortion. This can be prevented by restricting the maximum excursion of the trailing edge of the PWM pulse.

The spectrum of a naturally sampled PPM wave for a single tone modulating signal has a form similar to that of a PDM wave, with the only difference that it contains a component proportional to the derivative of the modulating signal in place of modulating components itself. Therefore, the PPM detection can be achieved by an LPF followed by an integrator. An alternative detection method is to convert PPM into PWM and then pass it



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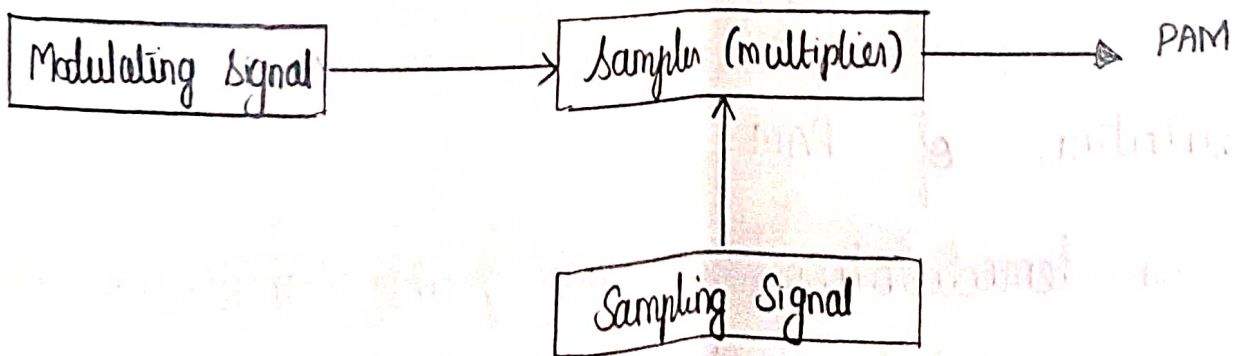
through an LPF. This provides a greater signal amplitude with less distortion in the receiver

PULSE AMPLITUDE MODULATION

Pulse amplitude modulation is the basic form of pulse modulation. In this modulation the signal is sampled at regular intervals and each sample is made proportional to the amplitude of the modulating signal.

Generation of PAM:

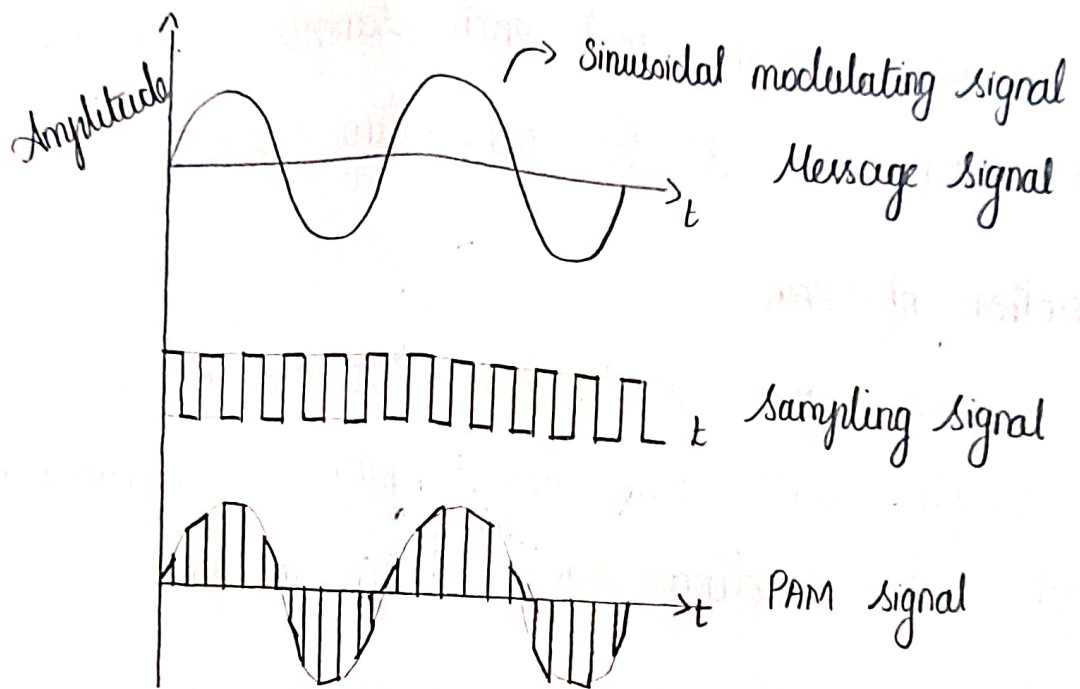
The figure shows the generation of PAM signal from the sampler which has two inputs. i.e. modulating signal and sampling signal or carrier pulse



Thus the amplitude of the signal is proportional to the modulating signal through which information is carried. This is pulse amplitude modulation signal.

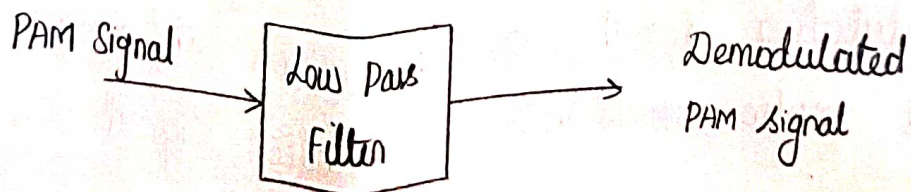
Fig 2 shows the spectrum of pulse amplitude modulated signal along with the message signal and the sampling waveform plotted in time domain.

Pulse modulation may be used to transmitting analog information such as continuous speech signal or data.



Demodulation of PAM:

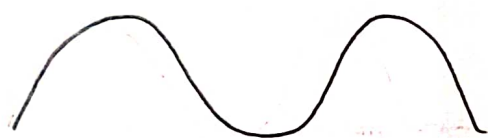
For demodulation of the pulse amplitude modulated signal, PAM is fed to the low pass filter



The low pass filter eliminates high frequency ripples
The sampling process generates the demodulated signal which has its
amplitude proportional to PAM signal at all time instant.

The signal is then applied to the inverting amplifier
to amplify its signal level to have the demodulated
output with almost equal amplitude with the
modulating signal.

The fig 3 shows modulated and demodulated PAM signal



Modulating signal



Modulated signal



Demodulated O/P

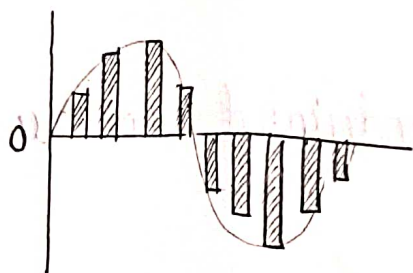
Types of sampling techniques of PAM :

There are two type of sampling techniques of
a signal using PAM

1. Flat top PAM
2. Natural PAM

Flat top PAM :

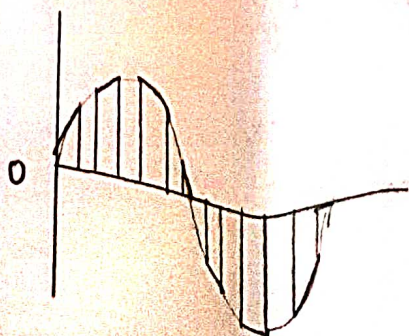
The amplitude of each pulse is directly proportional to modulating signal amplitude at the time of pulse occurrence. The amplitude of the signal cannot be changed with respect to analog signal to be sampled. The top of the amplitude remains flat.



Flat top PAM

Natural PAM :

The amplitude of each pulse is directly proportional to the modulating signal amplitude at the time of pulse occurrence. Then follows the amplitude of the pulse for the rest of the half cycle.



Natural PAM

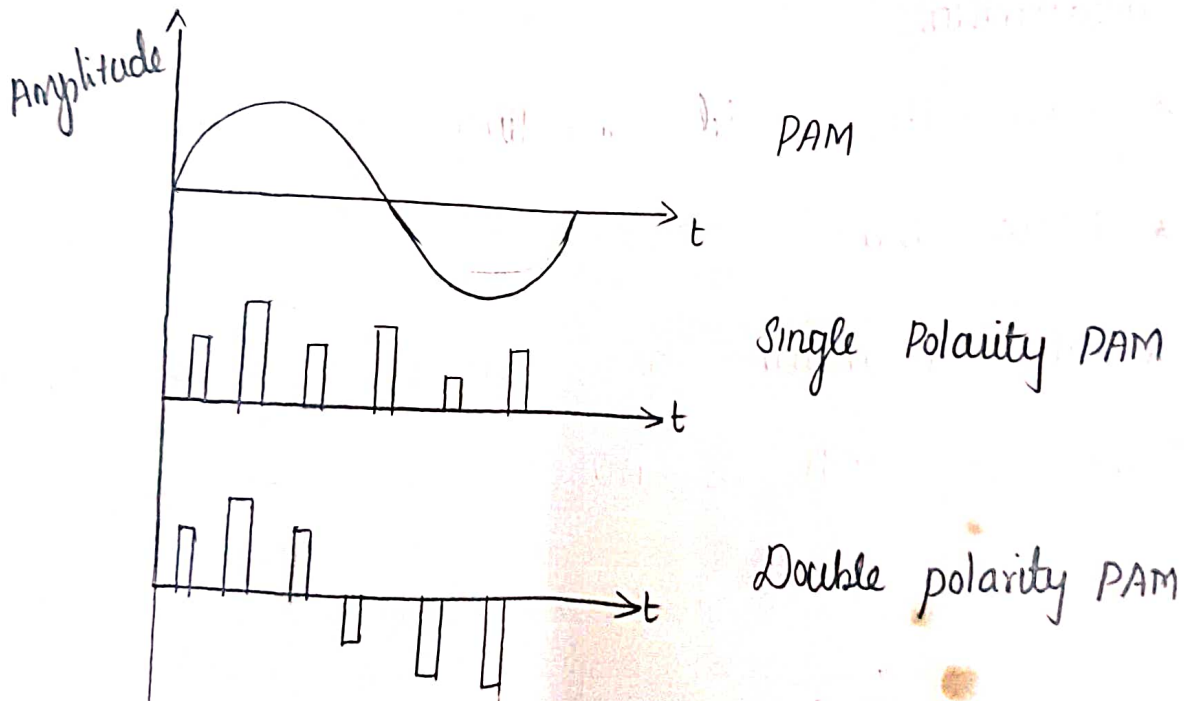
PAM:

SINGLE POLARITY PAM :

Single polarity PAM is a situation where a suitable fixed DC bias is added to the signal to ensure that all the pulses are positive

2. DOUBLE POLARITY PAM :

Double polarity PAM is a situation where the pulses are both positive and negative



Application of PAM:

- * It is used in Ethernet communication
- * It is used in Photo biology
- * It is used as an electronic driver for LED lighting
- * It is used in microcontrollers for generating the control signals

Advantages:

- * It is the simple process for both modulation and demodulation
- * Transmitter and receiver circuits are simple and easy to construct
- * PAM can generate other pulse modulation signals and can carry the message at the same time.

Disadvantages:

- * Bandwidth should be large for transmission PAM modulation
- * Noise will be great
- * Pulse amplitude signal varies so power required for transmission will be more

Library genius